1.

Q.1 **Statement 1:** When a solid sphere and a solid cylinder are allowed to roll down an inclined plane, the sphere will reach the ground first even if the mass and radius of the two bodies are different. Correct

Statement 2: The acceleration of the body rolling down the inclined plane is directly proportional to the radius of the rolling body. Incorrect

(1) Both Statement – 1 and Statement – 2 are true

(2✓) Statement – 1 is true and Statement – 2 is false

(3) Statement – 1 is False and Statement – 2 is true

(4) Both Statement – 1 and Statement – 2 are False

Sol.

Statement -1 $a = \dfrac{g\sin\theta}{1+\gamma}$

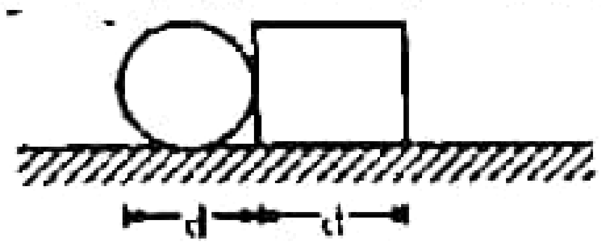
$t_{ss} < t_{cyl}$

$\gamma_{sp} = \dfrac{2}{5}$ $\gamma_{cy} = \dfrac{1}{2}$.

$t = \sqrt{\dfrac{2l}{a}} = \sqrt{\dfrac{2l(1+\gamma)}{g\sin\theta}}$

2.

Q.2 A square plate of edge d and a circular disc of diameter d are placed touching each other at the midpoint of an edge of plate as shown. Then center of mass of the combination will be (assume same mass per unit area for the two plates) -



(A) $\frac{2d}{2+\pi}$ left to the center of the disc

(B) $\frac{2d}{2+\pi}$ right to the center of the disc

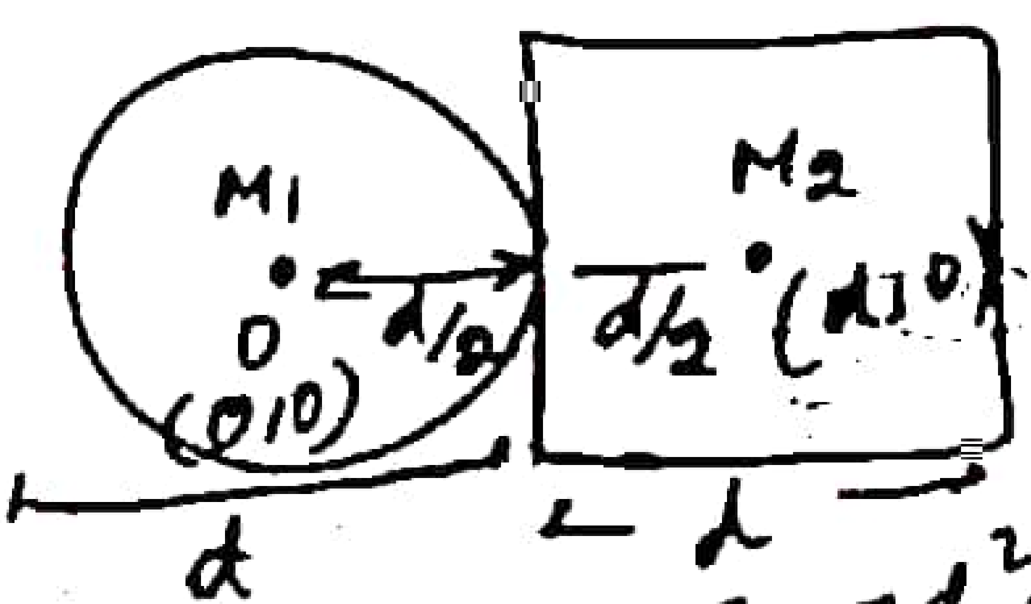
(C✓) $\frac{4d}{4+\pi}$ right to the center of the disc

(D) $\frac{4d}{4+\pi}$ left to the center of the disc

$$X_{com} = \frac{0 + M_2 d}{M_1 + M_2}$$

$$= \frac{\sigma d^2 . d}{\sigma \frac{\pi d^2}{4} + \sigma d^2}$$

$$= \frac{4d}{\pi + 4}$$



$M_1 = \sigma \times \pi R^2 = \frac{\pi d^2}{4}\sigma$

$M_2 = \sigma \times d^2$

3.

Q.3 The density of a linear rod of length L varies are $\rho = A + Bx$ where x is the distance from the left end. The position of the center of mass from the left end is -

(A) $\frac{2A+3BL^2}{3(2A+BL)}$ (B) $\frac{AL+3BL^2}{(2A+BL)}$ (C) $\frac{3AL+2BL^2}{(2A+BL)}$ (D) $\frac{3AL+2BL^2}{3(2A+BL)}$

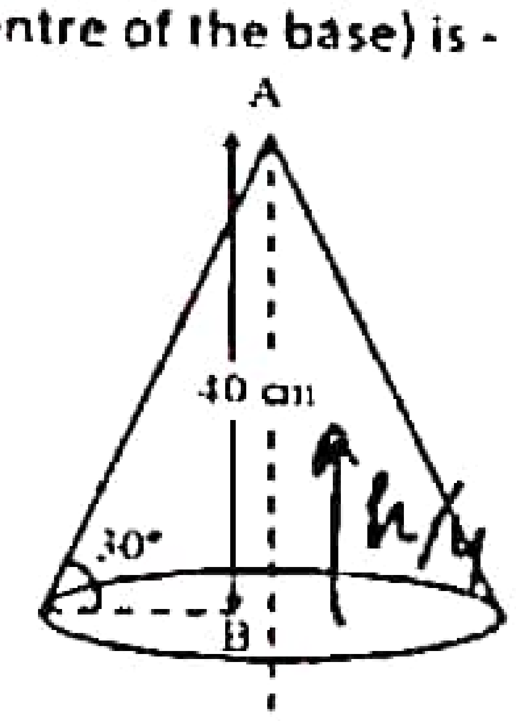
Sol^n

$$X_{com} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x \cdot \rho\,dx}{\int_0^L \rho\,dx} = \frac{\int_0^L (Ax+Bx^2)dx}{\int_0^L (A+Bx)dx}$$

$$X_{com} = \frac{\frac{AL^2}{2}+\frac{BL^3}{3}}{AL+\frac{BL^2}{2}} = \frac{3AL+2BL^2}{3(2A+BL)}$$

4.

Q.4 A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is -



$= \frac{40}{4} = 10\,cm$

(A) 20 cm (B) 10/3 cm (C) 20/3 cm (D) 10 cm

5.

Q.5 Two particles of equal mass have initial velocities $2\hat{i}$ ms^{-1} and $2\hat{j}$ ms^{-1}. First particle has a constant acceleration $(\hat{i} + \hat{j})$ ms^{-2} while the acceleration of the second particle is always zero. The centre of mass of the two particles moves in -

(A) circle (B) parabola (C) ellipse (D) straight line

Sol^n $\vec{u}_1 = 2\hat{i}$ $\vec{u}_2 = 2\hat{j}$

$a_1 = \hat{i} + \hat{j}$. $a_2 = 0$

$$\vec{u}_{com} = \frac{m\times 2\hat{i} + m\times 2\hat{j}}{2m}$$

$$\vec{u}_{com} = \hat{i} + \hat{j}$$

$$\vec{a}_{com} = \frac{m(\hat{i}+\hat{j}) + 0}{2m} = \frac{1}{2}(\hat{i}+\hat{j})$$

$$\vec{S} = \vec{u}_{com}t + \frac{1}{2}a_{com}t^2$$

$$\vec{S} = t\hat{i} + t\hat{j} + \frac{1}{4}(t^2\hat{i} + t^2\hat{j})$$

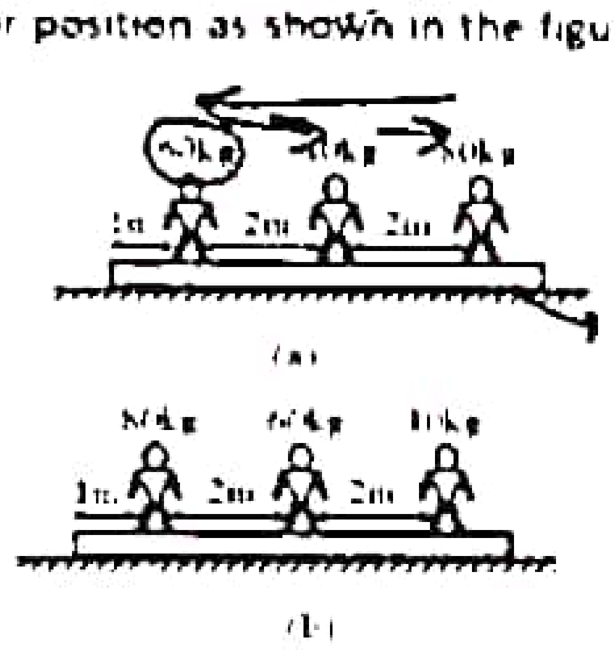
$$\vec{S} = \underbrace{\left(t + \frac{t^2}{4}\right)}\hat{i} + \underbrace{\left(t + \frac{t^2}{4}\right)}\hat{j}$$

$x = t + t^2/4$ $y = t + t^2/4$

$$\boxed{x = y}$$

6.

Q.6 Three boys are standing on a horizontal platform of mass 170 kg as shown in figure (a). They exchange their position as shown in the figure (b). Distance moved by the platform is :



$M = 170$ kg

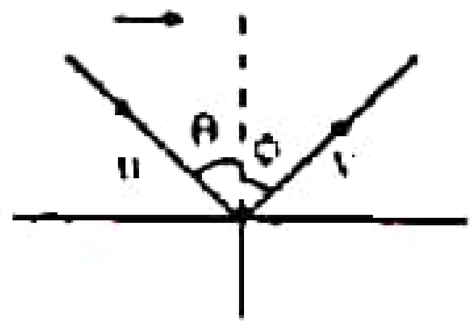
(A) 0.35m (B) 0.55m (C) 0.45m (D) 0.25m

$$X_{recoil} = \frac{-\Sigma m x_{rel}}{M + \Sigma m}$$

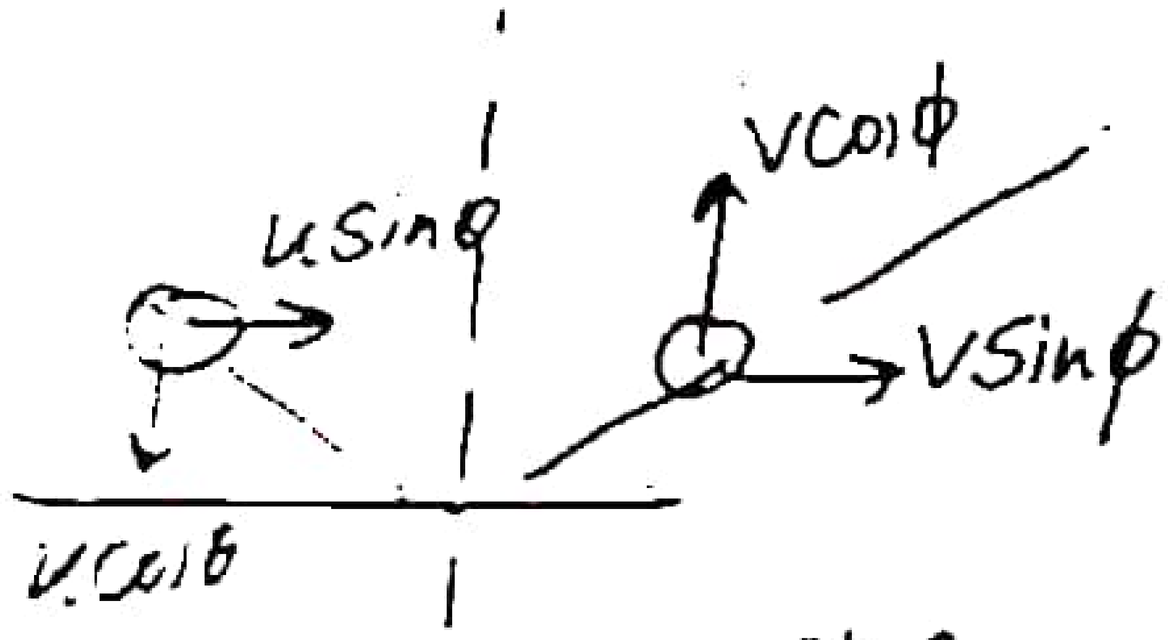
$$= -\frac{[60\times 2 + 40\times 2 - 80\times 4]}{170 + 180} = \frac{-[120 + 80 - 320]}{350}$$

$$= \frac{-[-120]}{360} = \frac{12}{35} \simeq 0.35m$$

7. A particle strikes a horizontal frictionless floor with a speed u at an angle θ with the vertical, and rebounds with a speed v, at an angle ϕ with the vertical. The coefficient of restitution between the particle and the floor is e. The angle ϕ is equal to -



(A) ... (B) $\tan^{-1}[e\tan\theta]$ (C) $\tan^{-1}\left[\frac{1}{e}\tan\theta\right]$ (D) $(1+e)\theta$



$\rightarrow\ v\sin\phi = u\sin\theta \quad -(1)$

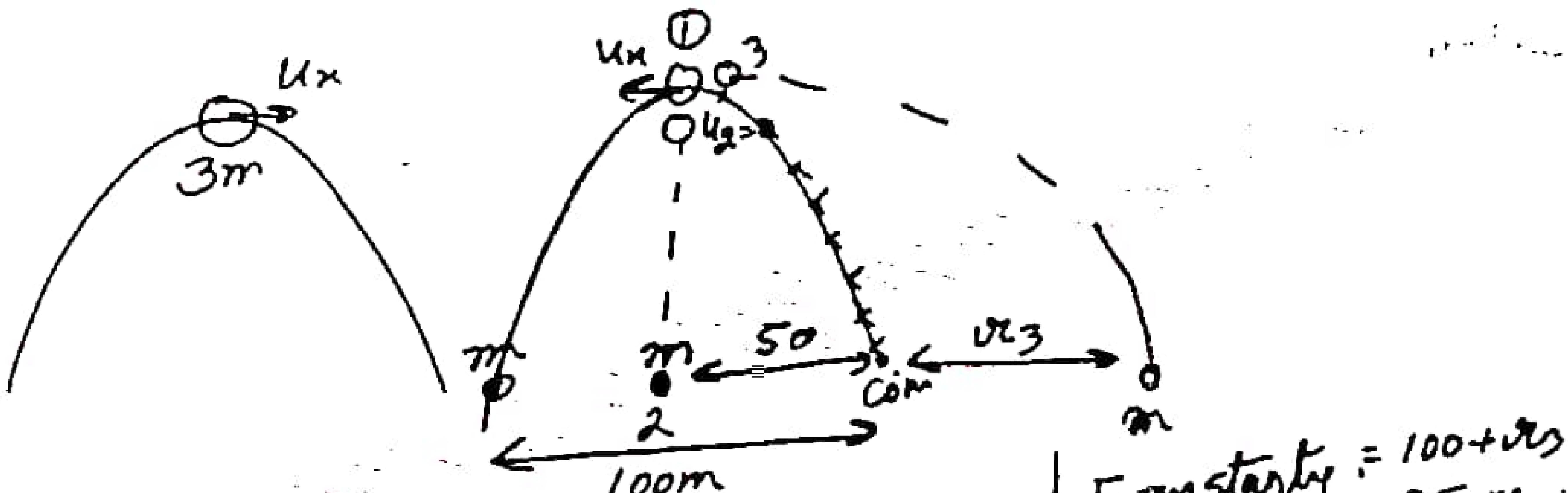
$v\cos\phi = e\,u\cos\theta \quad -(2)$

$\tan\phi = \frac{1}{e}\tan\theta$

$\phi = \tan^{-1}\left(\frac{1}{e}\tan\theta\right)$.

8. A projectile of mass 3 m explodes at highest point of its path. It breaks into three equal parts. One part retraces its path, the second one comes to rest. The range of the projectile was 100 m if no explosion would have taken place. The distance of the third part from the point of projection when it finally lands on the ground is -

(A) 100 m (B) 150 m (C) 250 m (D) 300 m



$100m + 50m = r_3 m$

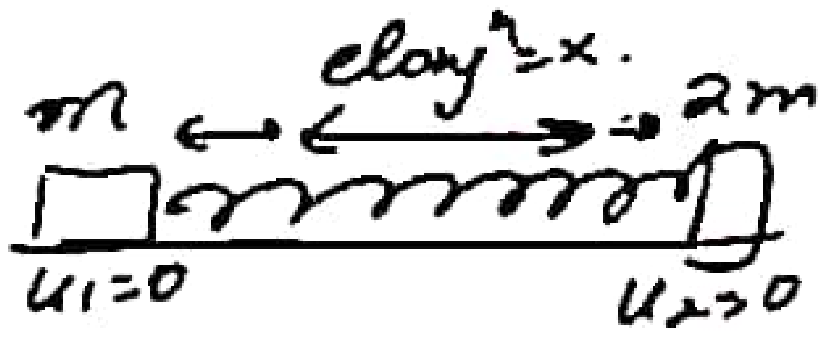
$r_3 = 150$

From starting $= 100 + r_3 = 250m$.

9.

Q.9 Two blocks A and B of mass m and 2 m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is -

(A) $\sqrt{\frac{3k}{2m}}\,x$ (B) $\left(\sqrt{\frac{2k}{3m}}\right)x$ (C) $\sqrt{\frac{2kx}{m}}$ (D) $\sqrt{\frac{3km}{2x}}$



$P_i = P_f$

$0 = mV_1 - 2mV_2$

$\boxed{V_1 = 2V_2}$

$\rightarrow K_i + U_i = K_f + U_f$

$0 + \frac{1}{2}kx^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2 + 0$

$kx^2 = 4mV_2^2 + 2mV_2^2$

$V_2 = \sqrt{\frac{kx^2}{6m}}$

$V_{rel} = V_1 + V_2 = 3V_2 = 3\sqrt{\frac{kx^2}{6m}} = \sqrt{\frac{9kx^2}{6m}} = \sqrt{\frac{3kx^2}{2m}}.$

10.

Q.15 Which one of the following statements is correct with reference to elastic collision between two bodies?

(A) Momentum and total energy are conserved but kinetic energy may be changed into some other form of energy

(B) Kinetic energy and total energy are both conserved but momentum is only if the two bodies have equal masses.

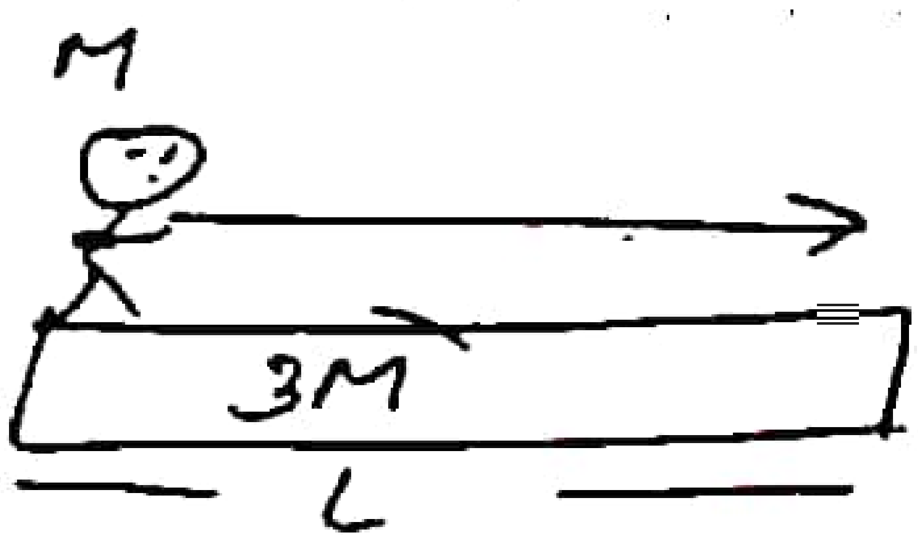
(C) Momentum, kinetic energy and total energy are all conserved

(D) Neither momentum nor kinetic energy need be conserved but total energy must be conserved

11.

Q.11 A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to the other end of the plank. If the mass of the plank is 3M, the distance that the man moves relative to the ground is -

(A) L/4 (B) 3L/4 (C) 2L/3 (D) L/3



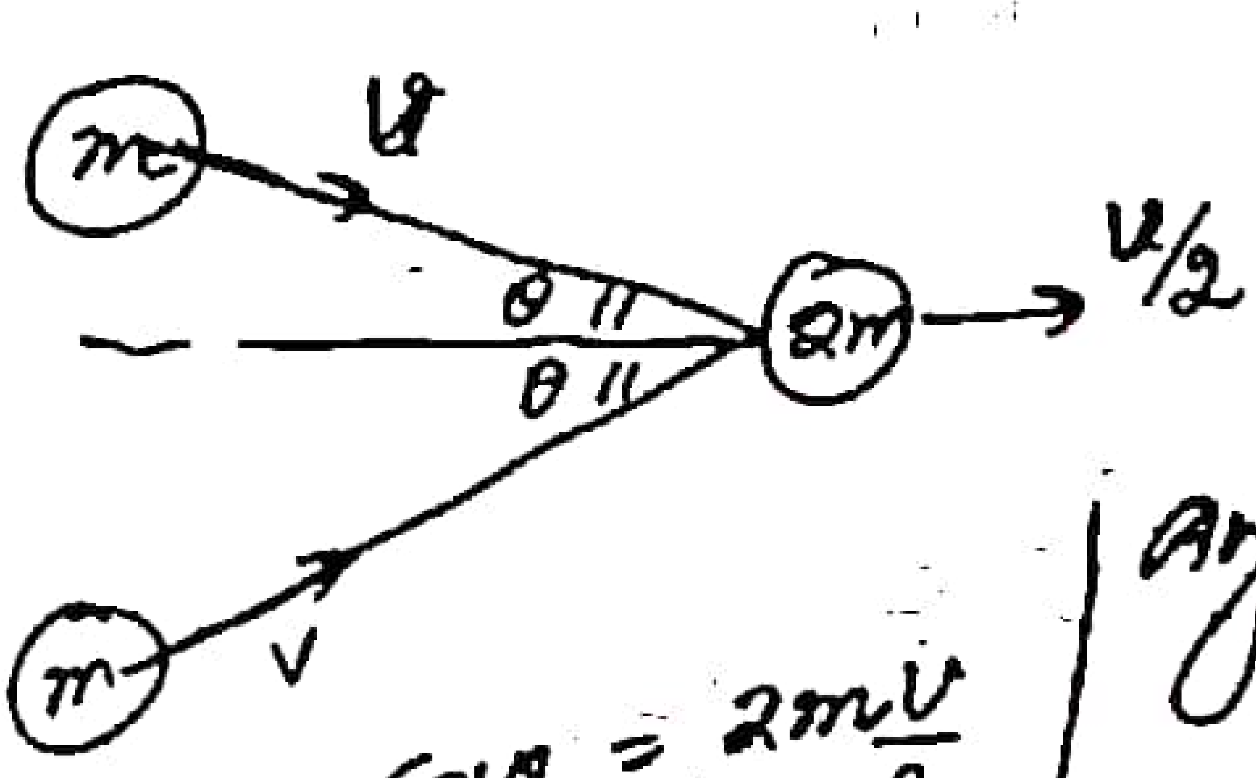
$$X_{Recoil} = -\frac{ML}{3M+M} = -\frac{L}{4}$$

$$X_{man/Gd} = X_{rel} + X_{Recoil} = L - \frac{L}{4} = \frac{3L}{4}$$

12.

Q 12 After a perfectly inelastic collision between two identical particles moving with same speed in different directions the speed of the particles becomes half the initial speed. The angle between velocities of the two before collision is -

(A) 60° (B) 45° (C) 120° (D) 30°



$$2mv\cos\theta = 2m\frac{v}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = 60}$$

Angle $= 2\theta = 120$

13.

Q.13 Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of 1 after collision ?

(A) 1/3 (B) 1/2 (C) ¼ (D) 1/6

$mu = mv + 2mv$

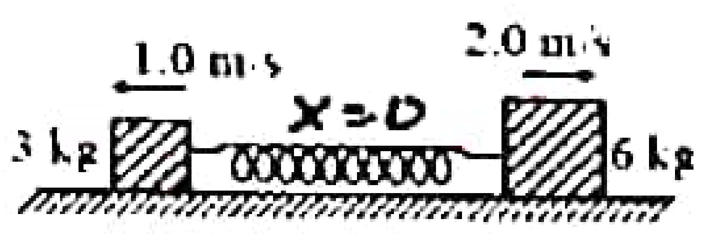
$\boxed{v = \frac{u}{3}}$

$e = \frac{V_2 - V_1}{u_1 - u_2}$

$e = \frac{2v - v}{u} = \frac{v}{u} = \frac{1}{3}$

14.

Q.14 Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be –



(A) 30 cm (B) 25 cm (C) 20 cm (D) 15 cm

Loss in K.E = Gain in P.E

$\frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(u_1-u_2)^2 = \frac{1}{2}kx^2$

$\frac{1}{2} \times \frac{18}{9} \times (9) = \frac{1}{2} \times 200 \times x^2$

$x = 0.3m$

$x = 30cm$

15.

Q.15 A bullet is fired from a gun. The force on the bullet is given by $F = 600 - (2 \times 10^5)\, t$. Here, F is in Newton and t in second. The force on the bullet becomes zero as soon as it leaves the barrel. The impulse imparted to the bullet is :

(A) 8 N-s (B) 0.9 N-s (C) 1.8 N-s (D) 2.4 N-s

$F = 600 - 2\times10^5 t$

$t = ?$ $F = 0$

$600 = 2\times10^5 t$

$t = \frac{300}{10^5}$

$\boxed{t = 3\times10^{-3}\,s}$

$\int_{p_i}^{p_f} dp = \int 600\,dt - 2\times10^5 t\,dt$

$\Delta p = (600t)_0^t - 10^5 (t^2)_0^t$

$= 600\times3\times10^{-3} - 10^5\times9\times10^{-6}$

$= 1.8 - 0.9$

$= 0.9$ N-s

16.

Q.16 A rigid body of mass M and radius R rolls without slipping on an inclined plane of inclination θ, under gravity. Match the type of body with magnitude of the force of friction

Column I		Column II	
(A)	For ring → s	(p)	$\frac{Mg\sin\theta}{2.5}$
(B)	For solid sphere → r	(q)	$\frac{Mg\sin\theta}{3}$
(C)	For solid cylinder → q	(r)	$\frac{Mg\sin\theta}{3.5}$
(D)	For hollow spherical shell	(s)	$\frac{Mg\sin\theta}{2}$

(1) A → s ; B → r ; C → q ; D → p

(2) A → q ; B → s ; C → p ; D → r

(3) A → p ; B → q ; C → r ; D → s

(4) A → r ; B → s ; C → p ; D → q

$f = \frac{\gamma Mg\sin\theta}{(1+\gamma)}$

$f_{Ring} = \frac{Mg\sin\theta}{2}$ $\gamma = 1$

$f_{s.s} = \frac{\frac{2}{5}Mg\sin\theta}{\frac{7}{5}}$ $\gamma = \frac{2}{5}$

$= \frac{2}{7}Mg\sin\theta = \frac{Mg\sin\theta}{3.5}$

$f_{cyl} = \frac{Mg\sin\theta}{3}$ $\gamma = \frac{1}{2}$

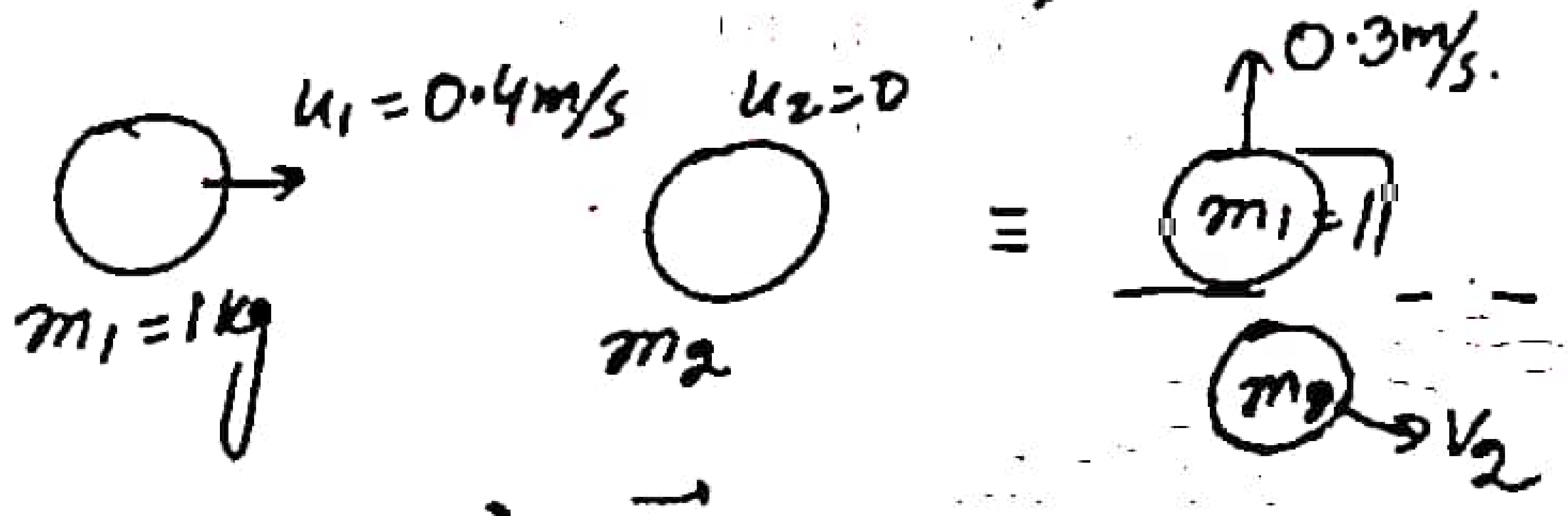
$f_{SH} = \frac{2}{5}Mg\sin\theta = \frac{Mg\sin\theta}{2.5}$ $\gamma = \frac{2}{3}$

Sol.

17.

Q.17 A ball of mass 1kg, moving with a velocity of 0.4 m/s, collides with another stationary ball. After the collision, the first ball moves with a velocity of 0.3 m/s in a direction making an angle of 90º with its initial direction. The momentum of second ball after collision will be (in kg-m/s) -

(A) 0.1 (B) 0.3 (C) 0.5 (D) 0.7



$$\vec{p}_i = \vec{p}_f$$

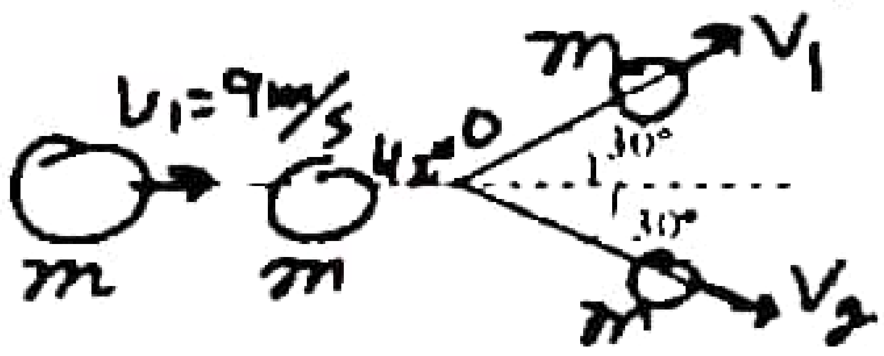
$$0.4\hat{i} + 0 = 0.3\hat{j} + \vec{p}_2$$

$$\vec{p}_2 = 0.4\hat{i} - 0.3\hat{j}$$

$$|\vec{p}_2| = 0.5 \text{ kg m/s}$$

18.

Q.18 A ball with velocity 9m/s collides with another similar stationary ball. After the collision the two balls move in directions making an angle of 30º with the initial direction (Fig.). The ratio of the speeds of balls after the collision will be-



(A) $\frac{v_1}{v_2} = 1$ (B) $\frac{v_1}{v_2} > 1$ (C) $\frac{v_1}{v_2} < 1$ (D) $\frac{v_1}{v_2} \neq 1$

$$p_{iy} = p_{fy}$$

$$0 = mV_1 \sin 30^\circ - mV_2 \sin 30^\circ$$

$$V_1 = V_2$$

$$\boxed{\frac{V_1}{V_2} = \frac{1}{1}}$$

19.

Q.19 As shown in figure A, B and C are identical balls B and C are at rest and, the ball A moving with velocity v collides elastically with ball B, then after collision –

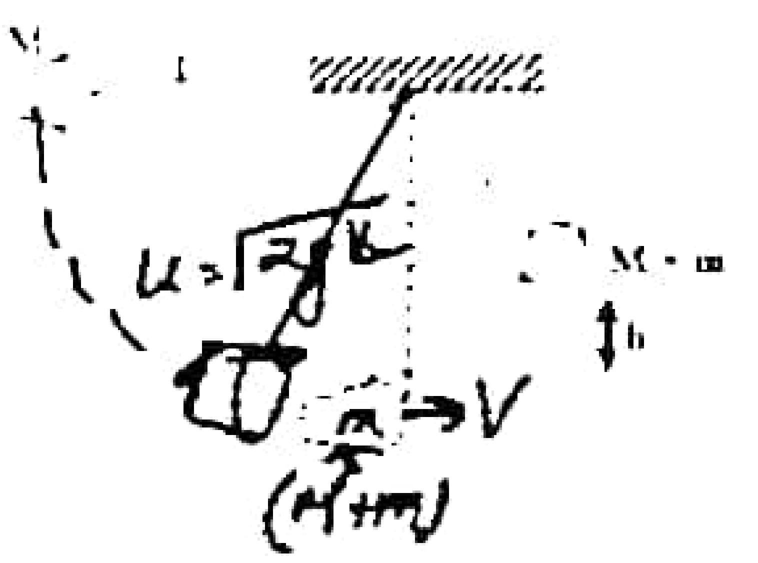
(A) All the three balls move with velocity v/2

(B) A comes to rest and (B + C) moves with velocity $v/\sqrt{2}$

(C) A moves with velocity v and (B + C) moves with velocity v

(D) A and B come to rest and C moves with velocity v

20. A small bucket of mass M is attached to a long inextensible cord of length L as shown in the figure. The bucket is released from its lowest position, the bucket scoops up mass m of water and swing to a height h. The height h is –



(A) $\frac{LM^2}{(M+m)^2}$ (B) $\frac{Lm^2}{(M+m)^2}$ (C) $\frac{LMm}{(M+m)^2}$ (D) $\frac{LMm}{(M-m)^2}$

$p_i = p_f$

$M \times u = (M+m)V$

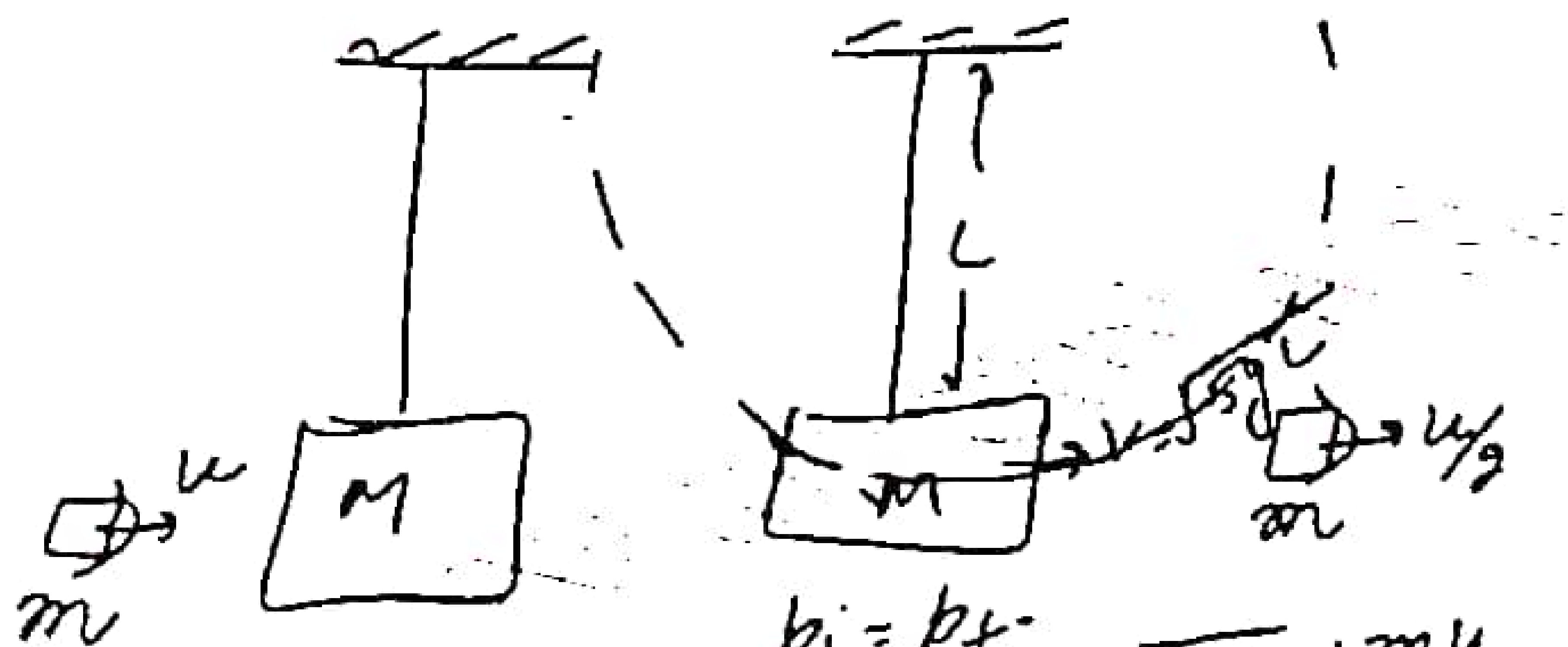
$M\sqrt{2gL} = (M+m)\sqrt{2gh}$

$$\boxed{\left(\frac{M}{M+m}\right)^2 L = h}$$

21.

Q.21 A bullet of mass m strikes a pendulum bob of mass M with velocity u. It passes through and emerges out with a velocity u/2 from bob. The length of the pendulum is ℓ. What should be the minimum value of u if the pendulum bob will swing through a complete circle?

(A) $\frac{2M}{m} \times \sqrt{5g\ell}$ (B) $\frac{M}{2m}\sqrt{5g\ell}$ (C) $\frac{2M}{m} \times \frac{1}{\sqrt{5g\ell}}$ (D) $\frac{M}{2m} \times \frac{1}{\sqrt{5g\ell}}$



$p_i = p_f$

$mu + 0 = M\sqrt{5gL} + \frac{mu}{2}$

$\frac{mu}{2} = M\sqrt{5gL}$

$\boxed{u = \frac{2M}{m}\sqrt{5gL}}$

22.

Q.22 A ball is dropped from a height h on the ground. If the coefficient of restitution is e, the height to which the ball goes up after it rebounds for the n^{th} time is-

(A) he^{2n} (B) $h e^{n}$

(C) $\frac{e^{2n}}{h}$ (D) $\frac{h}{e^{2n}}$

Sol.

$h_n = e^{2n} h$

23.

Q.23 **Statement–1:** When a girl jumps from a boat, the boat slightly moves away from the shore. Correct.

Statement–2: The total linear momentum of an isolated system remain conserved. Correct.

(A*) Both Statements are true.

(B) Statement–1 is True, Statement–2 is False

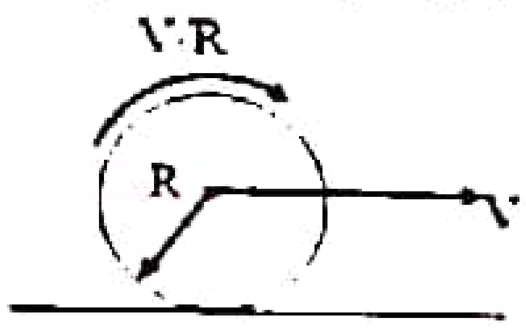
(C) Statement–1 is False, Statement–2 is False.

(D) Both Statements are False

Sol.

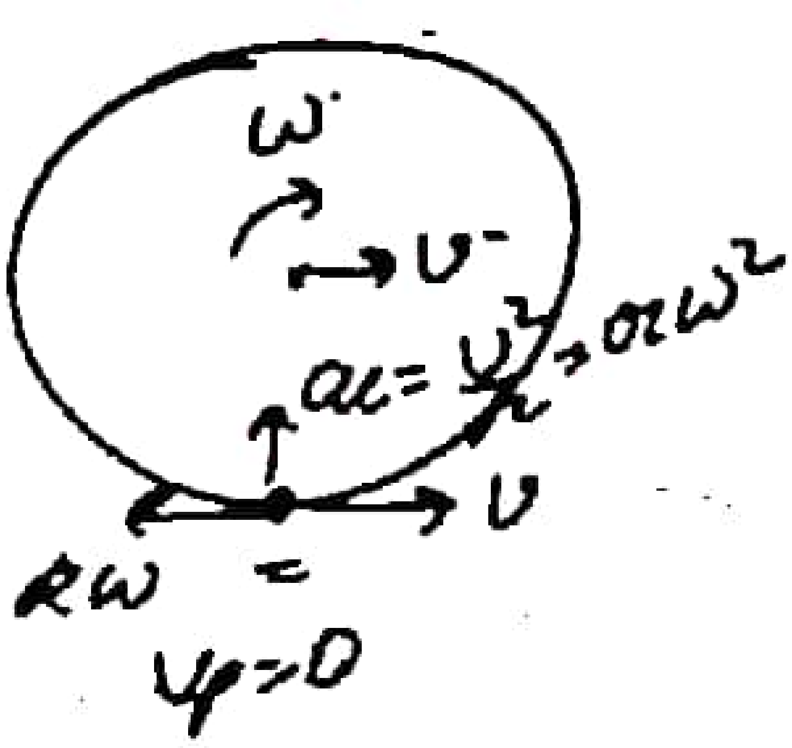
24.

Q.24 A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc.



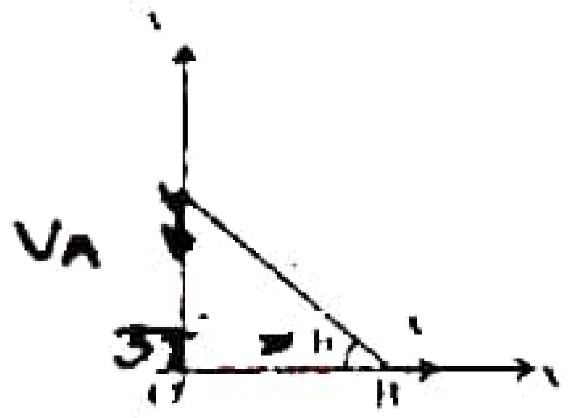
(A) Velocity is v, acceleration is zero (B) Velocity is zero, acceleration is zero

(C) Velocity is v, acceleration is $\frac{v^2}{R}$ (D*) Velocity is zero, acceleration is nonzero



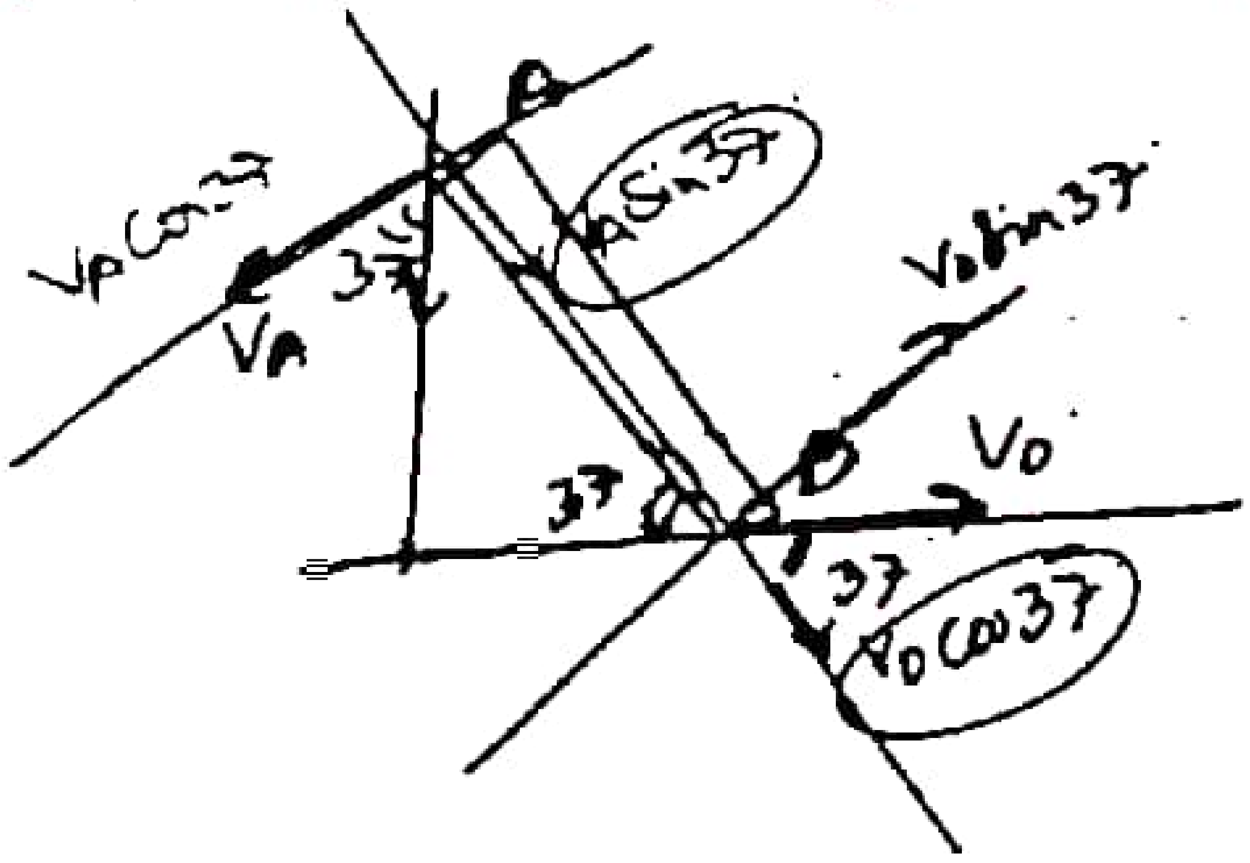
25.

Q.25 [illegible] the end B of the rod AB which makes angle θ with the floor [illegible] is pulled with a constant velocity v_0 as shown. The length of rod is ℓ. At an instant when $\theta = 37°$



~~(A)~~ velocity of end A is [illegible] (B) angular velocity of rod is [illegible]

(C) angular velocity of rod is constant (D) velocity of end A is constant



$$V_A \sin 37° = V_0 \cos 37°$$

$$V_A = \frac{4}{3} V_0$$

$$\omega = \frac{V_\perp}{\ell}$$

$$= \frac{V_0 \sin 37 + V_A \cos 37°}{\ell}$$

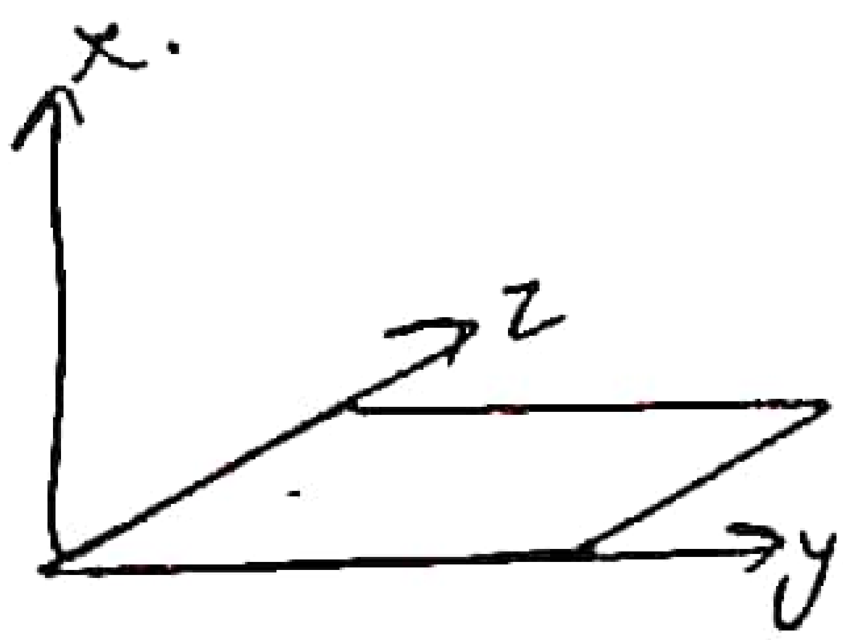
$$= \frac{5 V_0}{3 \ell}$$

26.

Q.26 A square plate is kept in yz-plane. Then according to perpendicular axis theorem -

(A) $I_z = I_x + I_y$ ~~(B*)~~ $I_x = I_y + I_z$ (C) $I_y = I_x + I_z$ (D) All

Sol.



$$I_x = I_y + I_z$$

27.

Q.27 A flywheel has moment of inertia 4 kg-m² and has kinetic energy of 200 J. Calculate the number of revolutions is makes before coming to rest if a constant opposing couple of 5N-m is applied to the flywheel -

(A) 12.8 rev (B) 24 rev (C*) 6.4 rev (D) 16 rev

$I = 4\,kg\text{-}m^2$

$K_i = 200\,J$

$K_f = 0$

$W = \Delta K$

$-\tau\theta = K_f - K_i$

$-5\times 2\pi n = 0 - 200$

$n = \frac{20\pi}{\pi^2}$

$= 2\pi$

$\simeq 6.4$ rev

28.

Q.28 The angular velocity of a body is $\vec{\omega} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and a torque $\vec{\tau} = \hat{i} + 2\hat{j} + 3\hat{k}$ acts on it. The rotational power will be-

(A*) 20 watt (B) 15 watt (C) $\sqrt{17}$ watt (D) $\sqrt{14}$ watt

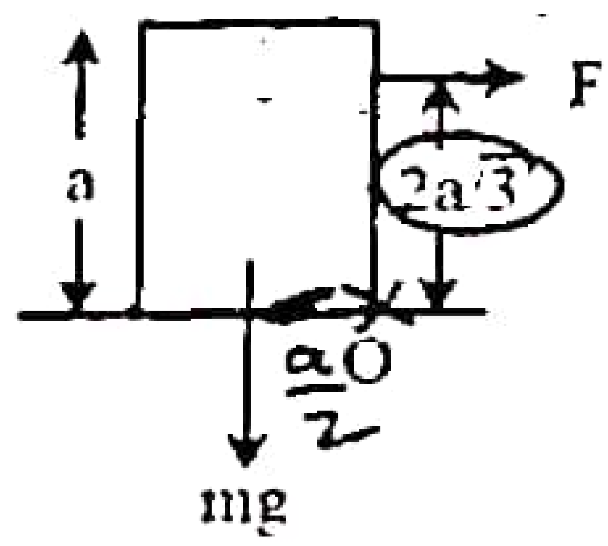
$P = \vec{\tau}\cdot\vec{\omega}$

$= 2 + 6 + 12$

$= 20$ watt

29.

Q.29 The minimum value of F for which the cube(a) begins to topple about an edge is–



(A) $\frac{2}{3}$ mg (B) $\frac{3}{4}$ mg (C) $\frac{1}{2}$ mg (D) mg

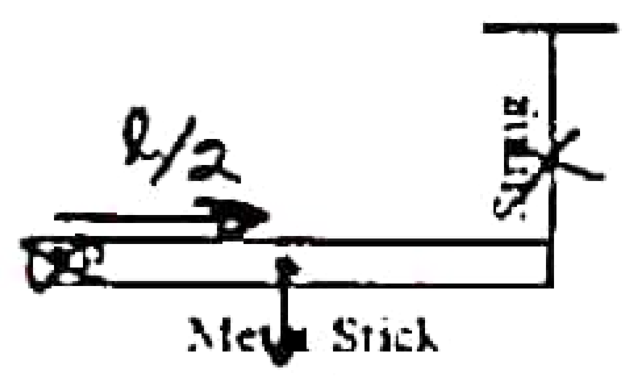
$\tau_{atF} \geq \tau_{Mg}$

$F\frac{2a}{3} \geq Mg\frac{a}{2}$

$\boxed{F \geq \frac{3}{4}Mg}$

30.

Q.30 A uniform meter stick of mass M is hinged at one end and supported in a horizontal position by a string attached to the other end as shown in figure. If the string is cut, then the initial angular acceleration of the stick is-



(A) g rad/s² (B) 3g rad/s² (C) 3g/2 rad/s² (D) 6g rad/s²

$\tau = I\alpha$

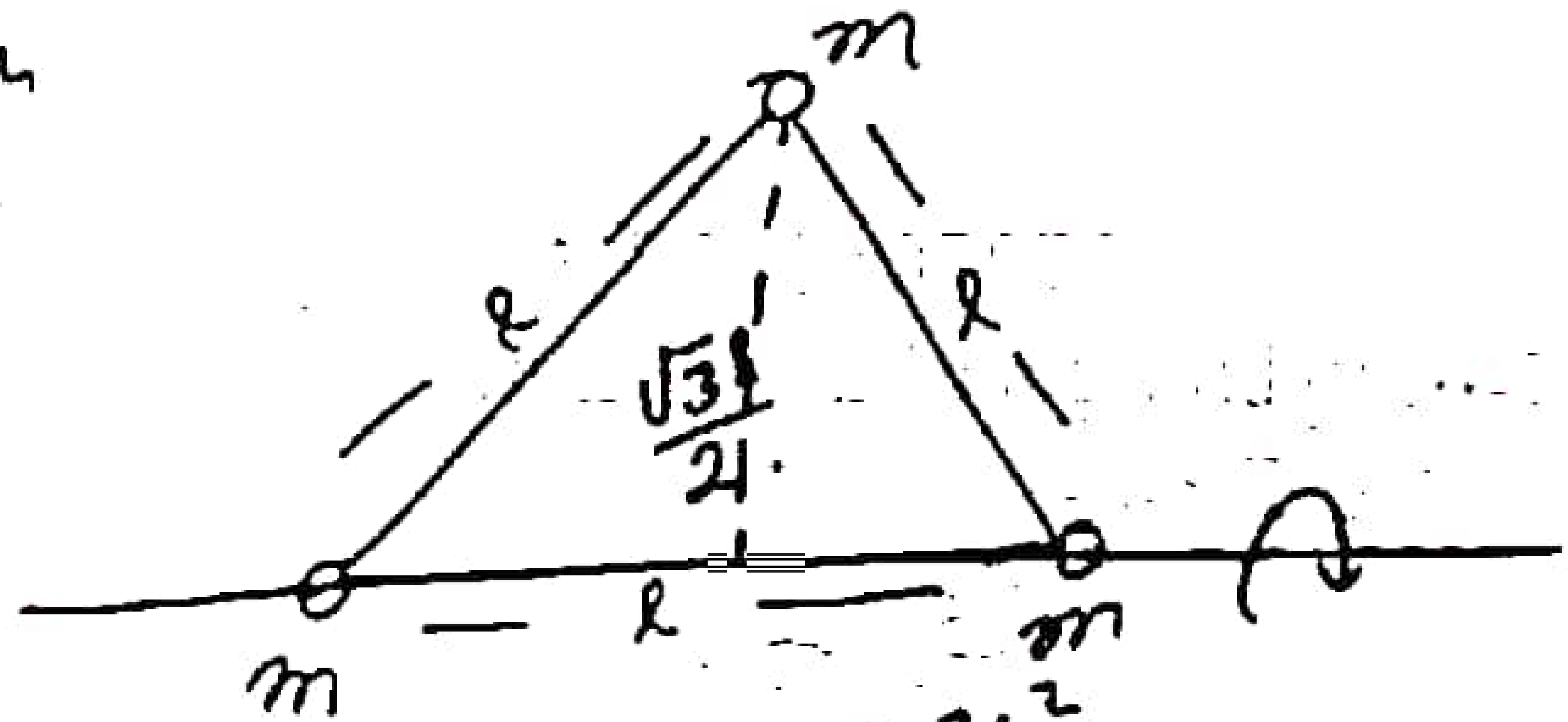
$Mg\frac{l}{2} = \frac{1}{3}Ml^2\alpha$

$\alpha = \frac{3g}{2l}$

31.
Q.31 Three point masses, each of mass m, are situated at the three corners of an equilateral triangle of each side ℓ. The moment of inertia of this system about one of the sides of the triangle will be -

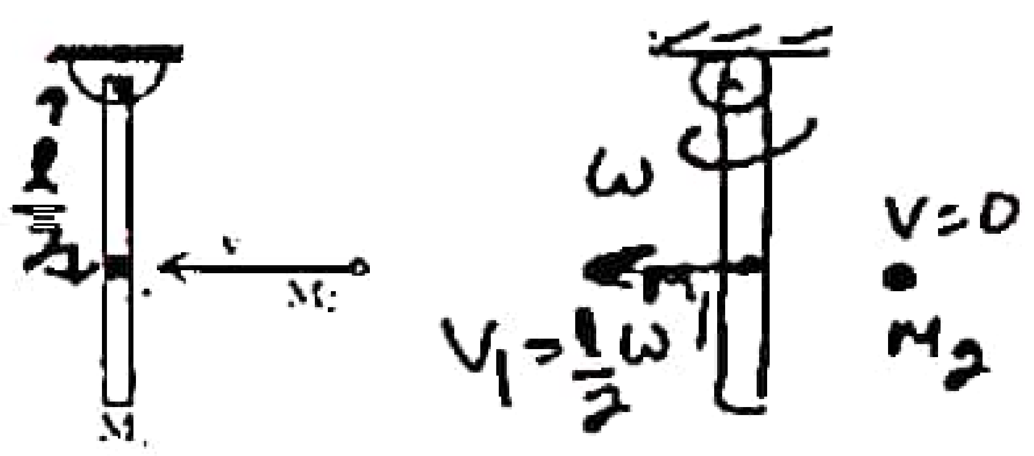
(A) $m\ell^2$ (B) $3m\ell^2$ (C) $\frac{3}{4}m\ell^2$ (D) $\frac{2}{3}m\ell^2$

Solⁿ



$$I = m \times \frac{3\ell^2}{4}$$

32.
Q.32 A uniform rod of mass M_1 is hinged at its upper end. A particle of mass M_2 moving horizontally strikes the rod at its mid point elastically. If the particle comes to rest after collision, the value of $\frac{M_1}{M_2}$ is –



(A) [illegible] (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

$$L_i = L_f$$

$$M_2 V \frac{\ell}{2} = I\omega$$

$$M_2 V \frac{\cancel{\ell}}{2} = \frac{1}{3} M_1 \ell^{\cancel{2}} \omega$$

$$\omega = \frac{3}{2}\frac{M_2}{M_1}\frac{V}{\ell}$$

$$e = \frac{V_1 - 0}{V - 0}$$

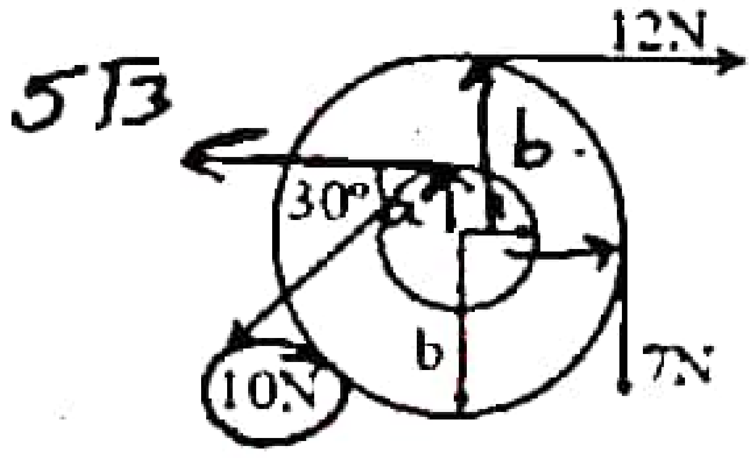
$$e = 1 = \frac{\ell}{2}\frac{\omega}{V}$$

$$1 = \frac{\cancel{\ell}}{2} \times \frac{3}{2}\frac{M_2}{M_1 \cancel{\ell}}$$

$$\boxed{\frac{M_1}{M_2} = \frac{3}{4}}$$

33.

Q.33 In the figure a = 6 cm and b = 20 cm. If the moment of inertia of the system is 3200 kg-m², its angular acceleration would be –



(A) 10^{-1}rad/s² (B) 10^{-2}rad/s² (C) 10^{-3}rad/s² (D) 10^{-4}rad/s²

$$\tau_N = 12b + 7b - 5\sqrt{3}a = I\alpha$$

$$19 \times 0.2 - 5\sqrt{3} \times 0.06 = 3200\alpha$$

$$3.8 - 0.3\sqrt{3} = 3200\alpha$$

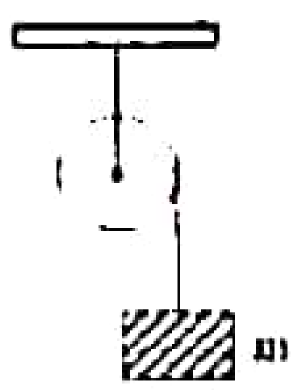
$$3.8 - 0.5 = 3200\alpha$$

$$\frac{3.3}{3200} = \alpha$$

$$\alpha \simeq 10^{-3}\ rad/s$$

34.

Q.34 In the following figure, a body of mass m is tied at one end of a light string and this string is wrapped around the solid cylinder of mass M and radius R. At the moment t = 0 the system starts moving. If the friction is negligible, angular velocity at time t would be-



(A) $\frac{mgR}{M+m}$ (B) $\frac{2Mgt}{(M+2m)}$ (C) $\frac{2mgt}{R(M-2m)}$ (D) $\frac{2mgt}{R(M+2m)}$

$$\alpha = \frac{mgR}{mR^2 + \frac{1}{2}MR^2}$$

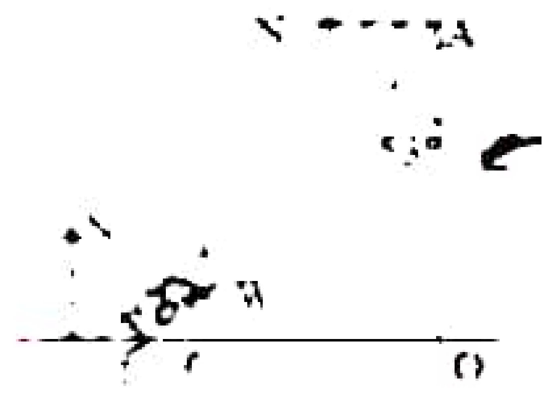
$$\alpha = \frac{2mg}{(2m+M)R}$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = \frac{2mgt}{(M+2m)R}$$

35.

35 A uniform ladder of mass 10 kg leans against a smooth vertical wall making an angle of 53° with it. The other end rests on a rough horizontal floor. Find the normal force N_1, N_2 and the frictional force 'f' that the floor exerts on the ladder-

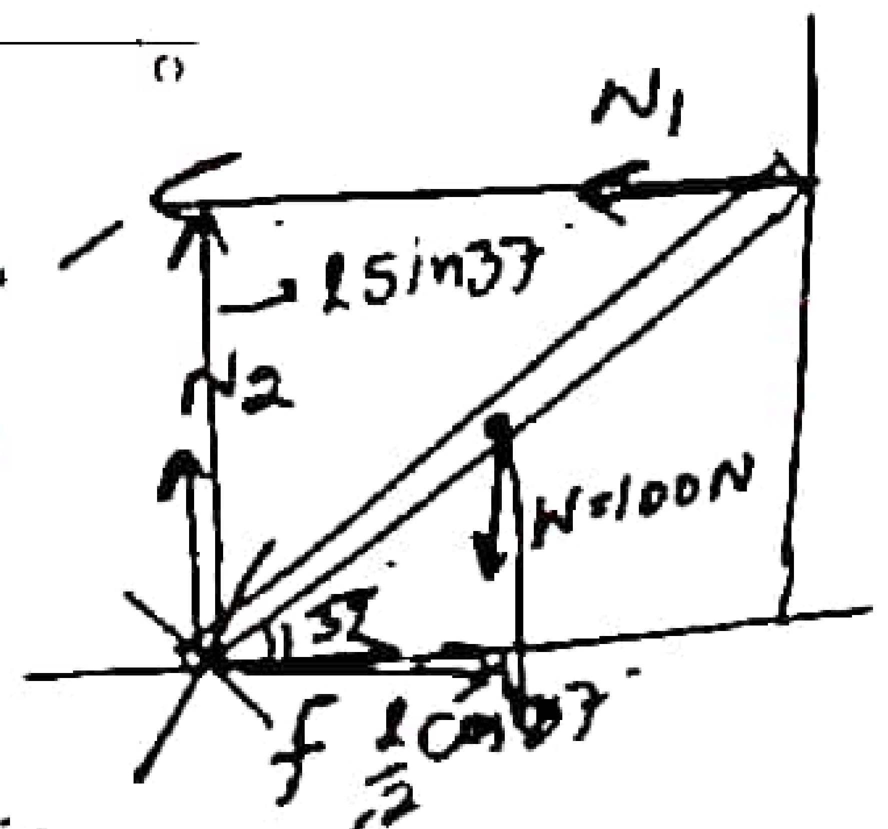


(A) 65 N, 65 N, 98 N

~~(B)~~ 65 N, 98 N, 65 N

(C) 98 N, 65 N, 65 N

(D) 65 N, 65 N, 65 N



X-axis $N_1 = f$

y-axis: $N_2 = 100N$

Torque balance

$N_1 l \sin 37° = 100 \frac{l}{2} \cos 37°$

$N_1 = 50 \times \frac{4}{3}$

$N_1 = 65N$

$f = 65$

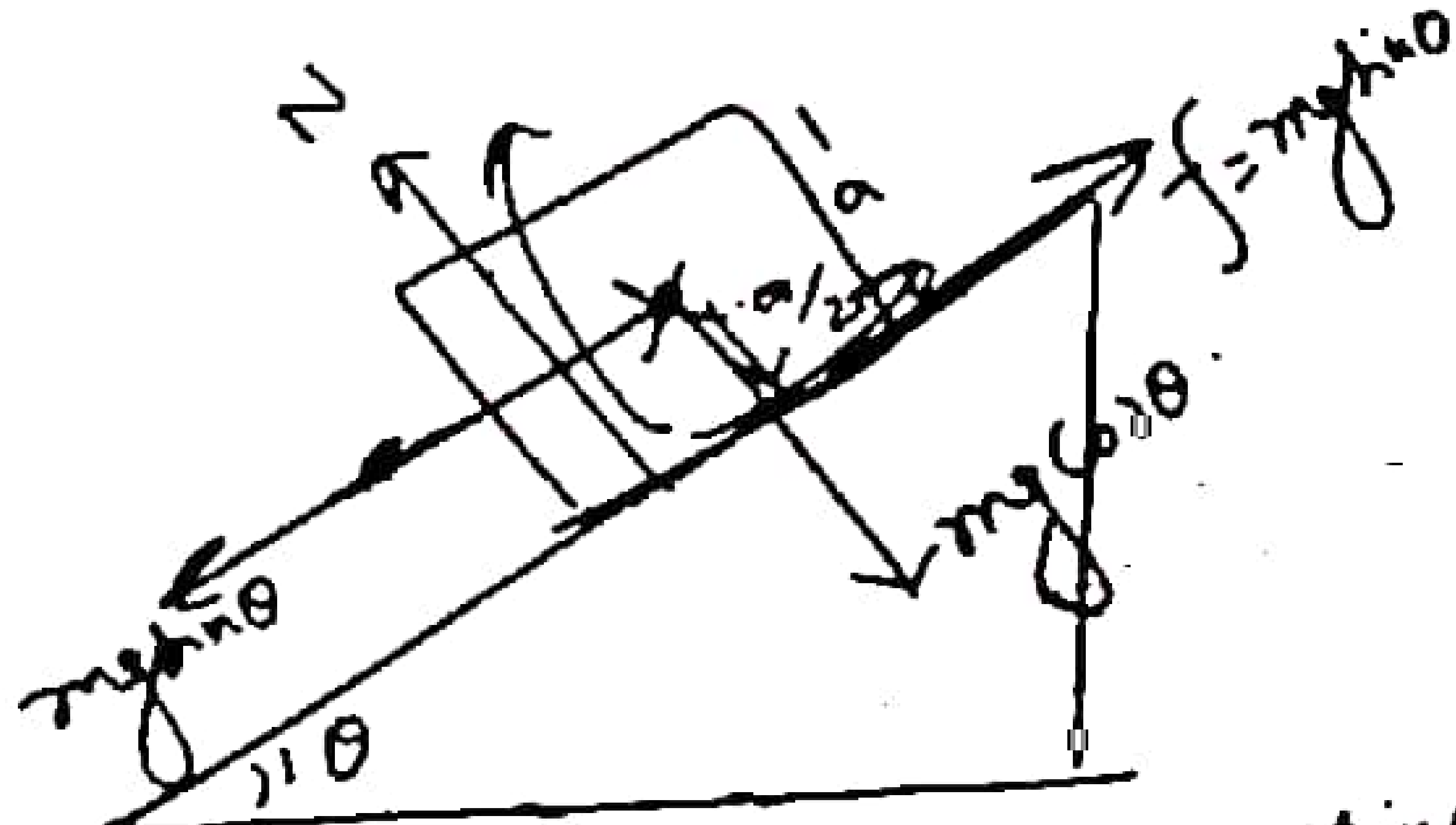
$N_1, N_2, f = 65, 100N, 65$

36.

36 A cubical block of mass m and edge a slides down a rough inclined plane of inclination θ with a uniform speed. Find the torque of the normal force acting on the block about its centre -

(A) mga sinθ (B) $\frac{1}{3}$ mga sinθ (C) $\frac{1}{4}$ mga sinθ ~~(D)~~ $\frac{1}{2}$ mga sinθ

Sol:



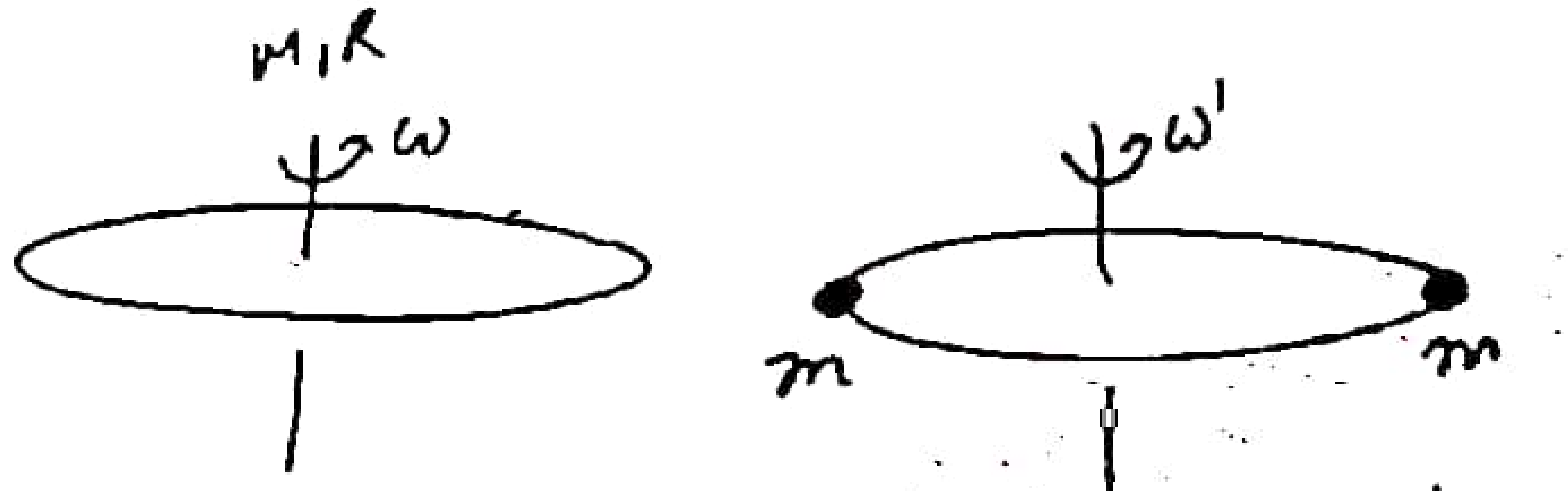
$\tau_N = \tau_f = mg\sin\theta \cdot \frac{a}{2}$

$\tau_N = mg\sin\theta \frac{a}{2}$

37.

37. A ring of mass M and radius R is moving in horizontal plane at angular speed ω about self axis. If two equal point masses are placed at the ends of any diameter. Find final angular speed of system -

(A) $\frac{M}{2m}\omega$ (B) $\frac{M}{M+2m}\omega$ (C) $\frac{m}{M+2m}\omega$ (D) none of above



$L_i = L_f$

$I_1\omega_1 = I_2\omega_2$

$MR^2\omega = (MR^2 + 2mR^2)\omega'$

$\omega' = \frac{M\omega}{(M+2m)}$

38.

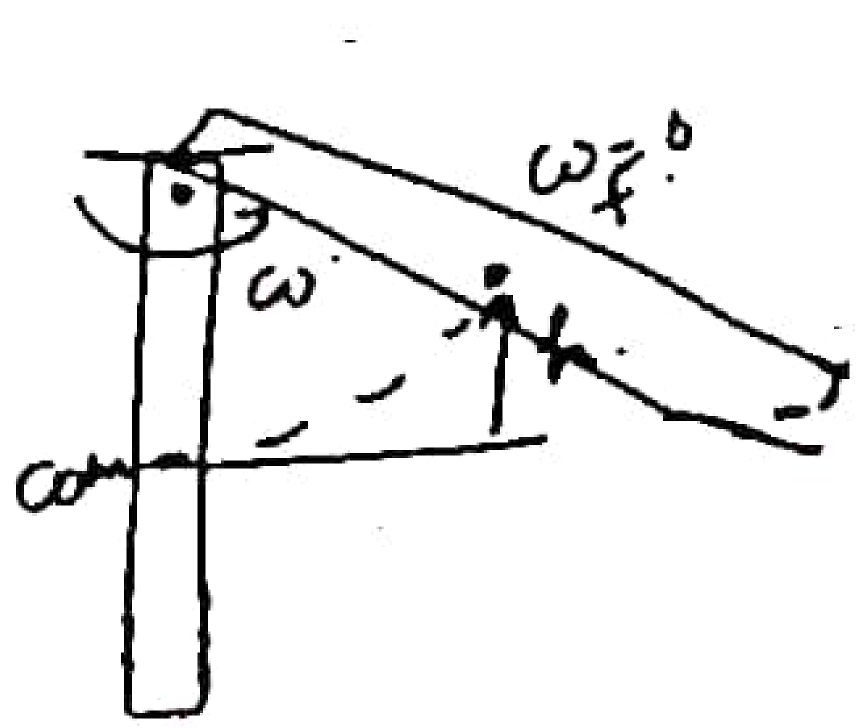
38. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of -

(A) $\frac{1}{6}\frac{\ell\omega}{g}$ (B) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (C) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (D) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$

$K_i = U_f$

$\frac{1}{2}I\omega^2 = mgh$

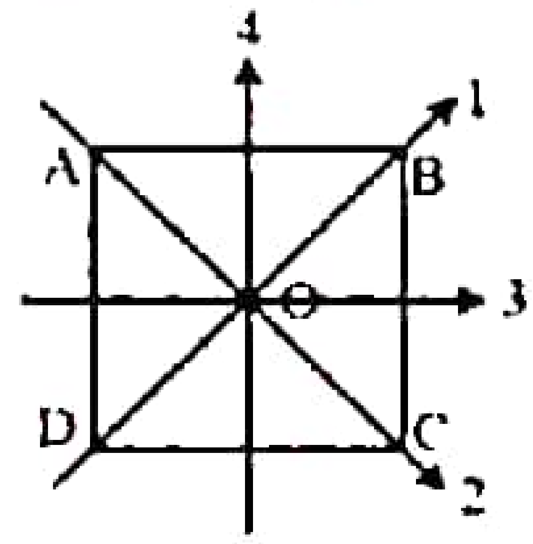
$\frac{1}{2}\times\frac{1}{3}m\ell^2\omega^2 = mgh$



$h = \frac{\ell^2\omega^2}{6g}$

39.

39. The moment of inertia of a thin square plate ABCD (figure) of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is:



(A) $I_1 + I_2$ (B) $I_3 + I_4$ (C) $I_1 + I_3$ (D) All of these

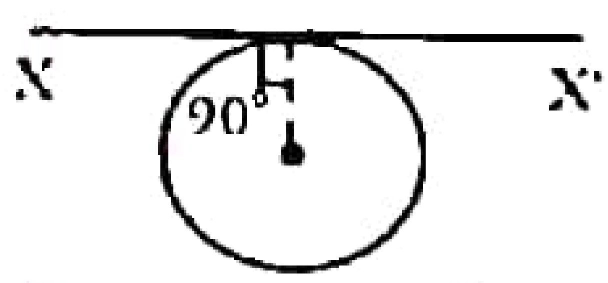
$I_O = I_1 + I_2$

$I_O \neq I_3 + I_4$

$I_O = I_1 + I_3$

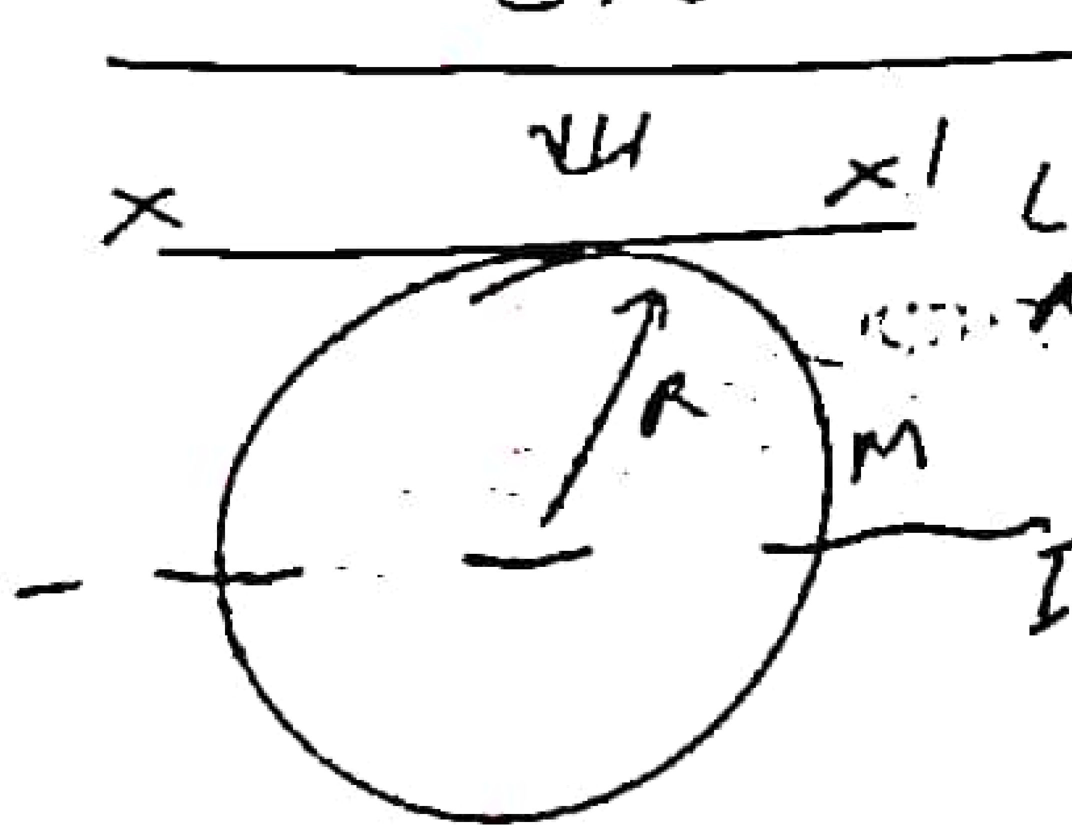
40.

40. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is :



(A) $\frac{\rho L^3}{8\pi^2}$ (B) $\frac{\rho L^3}{16\pi^2}$ (C) $\frac{5\rho L^3}{16\pi^2}$ (D) $\frac{3\rho L^3}{8\pi^2}$

$L, \rho \quad M = \rho L$



$L = 2\pi R$

$R = \frac{L}{2\pi}$

$I_{com} = \frac{1}{2} MR^2$

$I_{xx'} = \frac{1}{2} MR^2 + MR^2$

$= \frac{3}{2} MR^2$

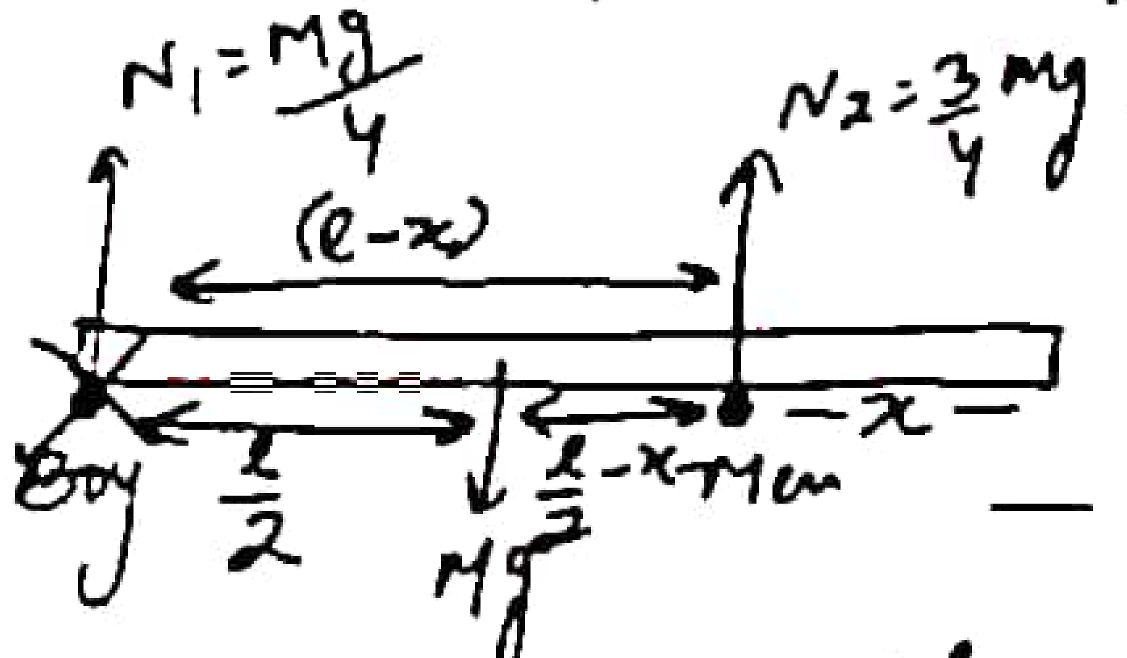
$= \frac{3}{2} \times \rho L \times \frac{L^2}{4\pi^2}$

$= \frac{3\rho L^3}{8\pi^2}$

41.

41. A boy and a man carry a uniform rod of length L horizontally in such a way that boy gets (1/4)th load. If the boy is at one end of the rod, the distance of the man from the other end is -

(A) $\frac{L}{3}$ (B) $\frac{L}{4}$ (C) $\frac{2L}{3}$ (D) $\frac{3L}{4}$



$N_2(l-x) = Mg \times \frac{l}{2}$

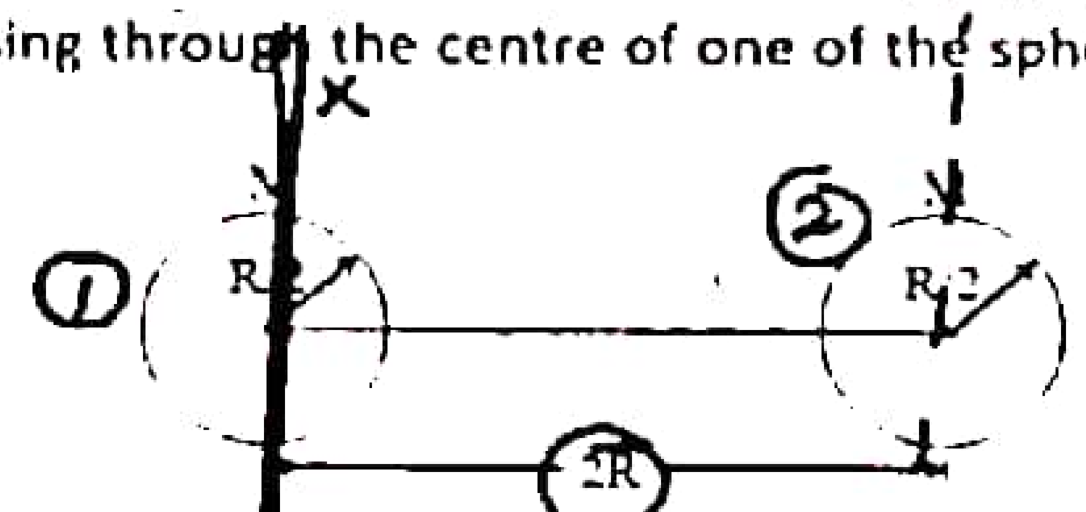
$\frac{3}{4}(l-x) = \frac{l}{2}$

$3l - 3x = 2l$

$x = \frac{l}{3}$

42.

42. Two spheres each of mass M and radius $\frac{R}{2}$ are connected with a massless rod of length 2R as shown in the figure. What will be the moment of inertia of the system about an axis passing through the centre of one of the spheres and perpendicular to the rod -



(A) $\frac{21}{5}MR^2$ (B) $\frac{2}{5}MR^2$ (C) $\frac{5}{2}MR^2$ (D) $\frac{5}{21}MR^2$

$I_{2/x} = I_2 + Mh^2$

$= \frac{2}{5} \times M \times \frac{R^2}{4} + M \times 4R^2$

$I_{2x} = \frac{MR^2}{10} + 4MR^2$

$I_{1x} = \frac{2}{5} M \times \frac{R^2}{4}$

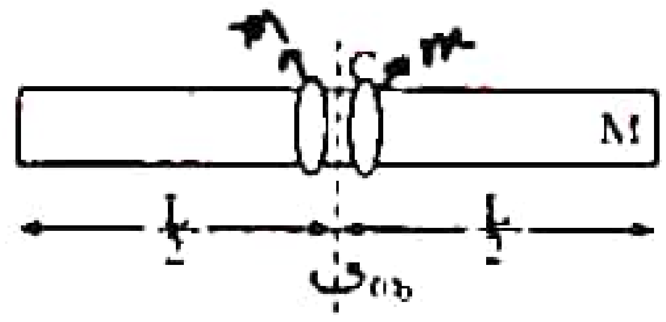
$= \frac{MR^2}{10}$

$I_T = \frac{MR^2}{5} + 4MR^2$

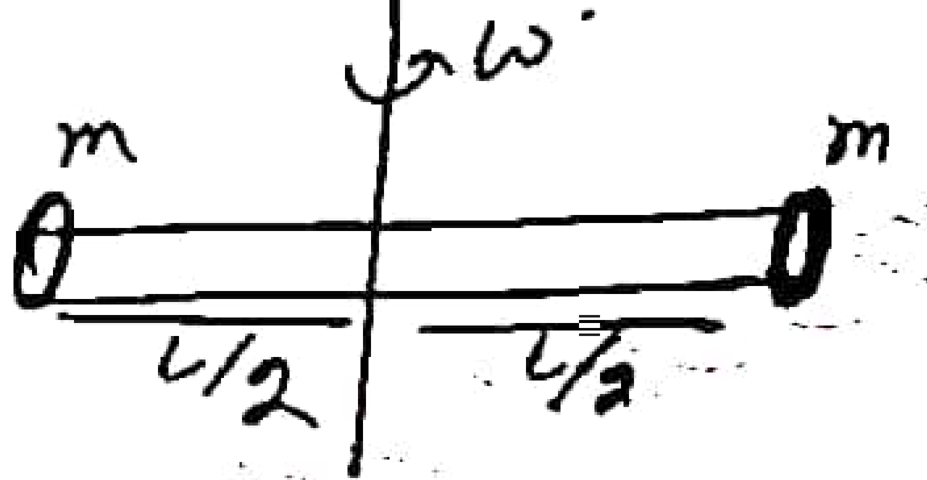
$= \frac{21}{5} MR^2$

43.

41. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about an axis perpendicular to rod and passing through the mid-point of rod. There are no external forces. When the beads reach the ends of the rod the angular velocity of the system is -



(A) $\frac{M}{M+3m}\omega_0$ (B) $\frac{M}{M+6m}\omega_0$ (C) $\frac{M+6m}{M}\omega_0$ (D) ω_0



$L_i = L_f$

$I_1\omega_1 = I_2\omega_2$

$\frac{1}{12}ML^2\omega_0 = \left[\frac{1}{12}ML^2 + 2m\frac{L^2}{4}\right]\omega'$

$\omega' = \frac{M\omega_0}{M+6m}$

44.

44. A disc of metal is melted to recast in the form of a solid sphere of same radius. The moment of inertia about a vertical axis passing through the centre would-

(A) decrease

(B) increase

(C) remains same

(D) nothing can be said

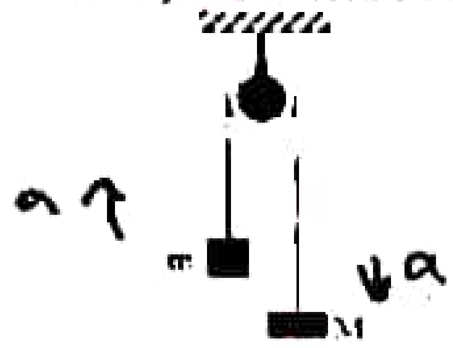
Sol.

$I_D = \frac{1}{2}MR^2 = 0.5MR^2$ $I_S = \frac{2}{5}MR^2$

$I_S = 0.4MR^2$

45.

45. Two masses m and M (>m) are joined by a light string passing over a smooth light pulley. The centre of mass of the system moves with an acceleration –



(A) $g\frac{M-m}{M+m}$ downward $\quad$ ~~(B)~~ $g\left(\frac{M-m}{M+m}\right)^2$ downward

(C) $g\frac{M-m}{M+m}$ upward if M < m $\quad$ (D) Zero

$$a = \frac{(M-m)g}{(M+m)}$$

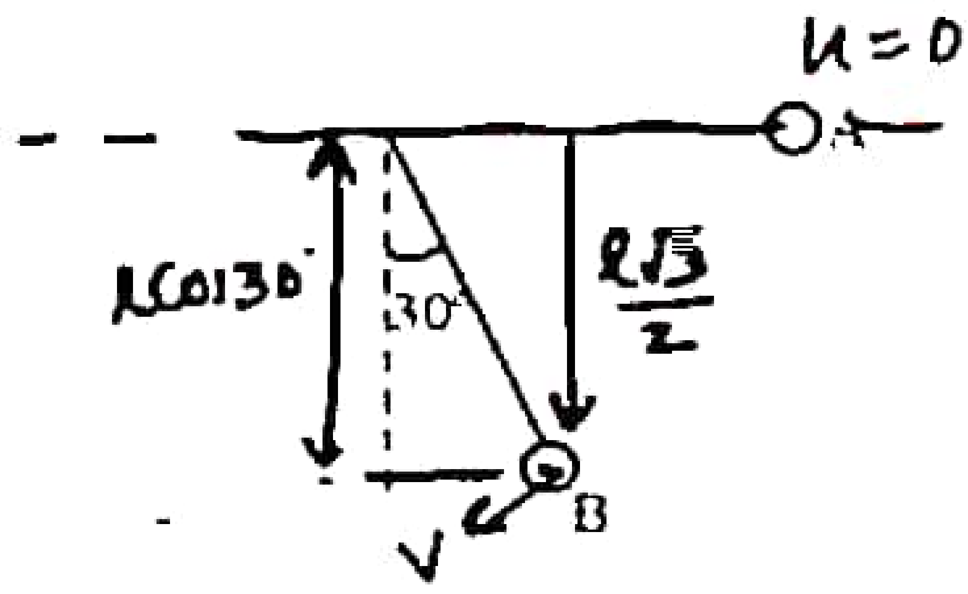
$$a_{com} = \frac{-Ma + ma}{M+m}$$

$$= -\left(\frac{M-m}{M+m}\right) \times \left(\frac{M-m}{M+m}\right) g$$

$$a_{com} = -\left[\frac{M-m}{M+m}\right]^2 g$$

46.

46. A simple pendulum is released from A as shown. If m and ℓ represent the mass of the bob and length of the pendulum, the gain in kinetic energy at B is:



(A) $\frac{mg\ell}{2}$ $\quad$ (B) $\frac{mg\ell}{\sqrt{2}}$ $\quad$ ~~(C)~~ $\frac{\sqrt{3}}{2}mg\ell$ $\quad$ (D) $\frac{2}{\sqrt{3}}mg\ell$

$$K_i + U_i = K_f + U_f$$

$$\text{Gain in K.E} = K_f - K_i = U_i - U_f$$

$$= 0 - \left[-Mg\frac{\ell\sqrt{3}}{2}\right]$$

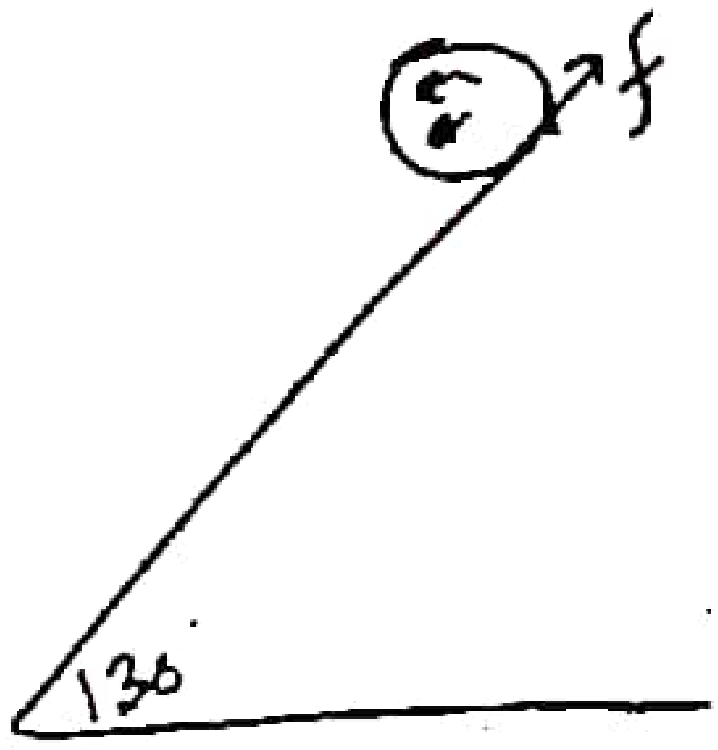
$$= Mg\ell\frac{\sqrt{3}}{2}$$

47.

47. A uniform disc of mass m and radius R is rolling down a rough inclined plane which makes an angle 30° with the horizontal. If the coefficients of static and kinetic friction are each equal to μ and the only force acting are gravitational and frictional, then the magnitude of the frictional force acting on the disc is -

(A) (mg/3) upwards (B) (mg/3) downwards

(C*) (mg/6) upwards (D) (mg/6) downwards



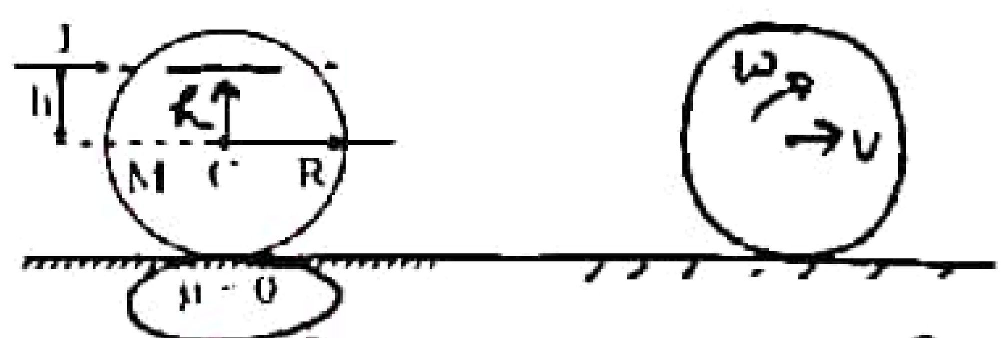
$$f = \frac{\gamma Mg\sin\theta}{1+\gamma}$$

$$f = \frac{\frac{1}{2}Mg\sin 30°}{\frac{3}{2}}$$

$$f = \frac{Mg}{6} \text{ [upward]}$$

48.

48. A solid sphere of mass M and radius R, is placed on a smooth horizontal surface. It is given a horizontal impulse J at a height h above the Centre of mass and sphere starts rolling then, the value of h and speed of Centre of mass are -



(A*) $h = \frac{2}{5}R$ and $v = \frac{J}{M}$ (B) $h = \frac{2}{5}R$ and $v = \frac{2}{5}\frac{J}{M}$

(C) $h = \frac{7}{5}R$ and $v = \frac{7}{5}\frac{J}{M}$ (D) $h = \frac{7}{5}R$ and $v = \frac{J}{M}$

Sol For Rolling $v_P = 0$

$\Rightarrow v = R\omega$

Linear Impulse

$J = p_f - p_i = Mv - 0$

$v = \frac{J}{M}$ (1)

Angular Impulse

$Jr = L_f - L_i$ (Abt COM)

$Jh = I\omega - 0$

$Jh = \frac{2}{5}MR^2\omega$

$R\omega = \frac{5Jh}{2MR}$ (2)

$v = R\omega$

$\frac{J}{M} = \frac{5}{2}\frac{Jh}{MR}$

$\boxed{h = \frac{2}{5}R}$

49.

49. If net force on a system of particles is zero, then

$F_{net} = 0$

$a_{COM} = 0$

$V_{COM} = \text{Const}$ $V_{COM} = 0$

	Column I		Column II
(A)	Acceleration of centre of mass → q	(p)	Constant
(B)	Velocity of centre of mass p, r	(q)	Zero
(C)	Momentum of centre of mass p, r	(r)	May be zero
(D)	Velocity of an individual particle of the system - r, s	(s)	May be constant

(1*) (A) → q; (B) → p,r; (C) → p,r; (D) → r,s

(2) (A) → s; (B) → p,s; (C) → q,r; (D) → q

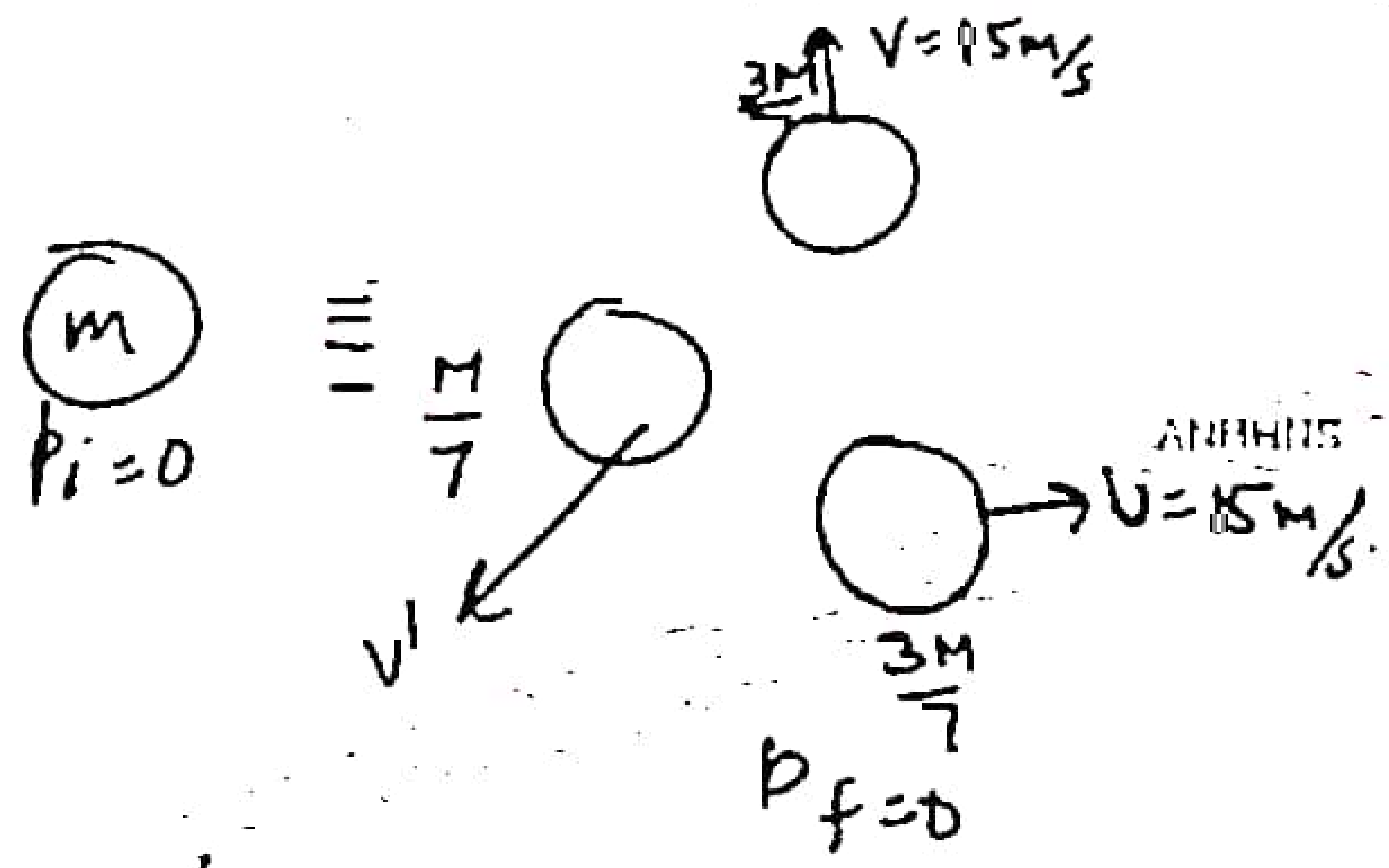
(3) (A) → p; (B) → q,r; (C) → p,s; (D) → r,s

(4) (A) → r; (B) → p,r; (C) → ~~q~~; (D) → p,s

50.

50. A stationary body of mass m gets exploded in 3 parts having mass in the ratio of 1: 3: 3. Its two fractions having equal mass moving at right angle to each other with velocity of 15 m/sec. Then the velocity of the third body is

(A*) $45\sqrt{2}$m/sec (B) 5m/s (C) $5\sqrt{2}$m/sec (D) none of these



$\frac{M}{7} v' = \sqrt{2}\, \frac{3M}{7} \times 15$

$v' = 45\sqrt{2}$ m/s