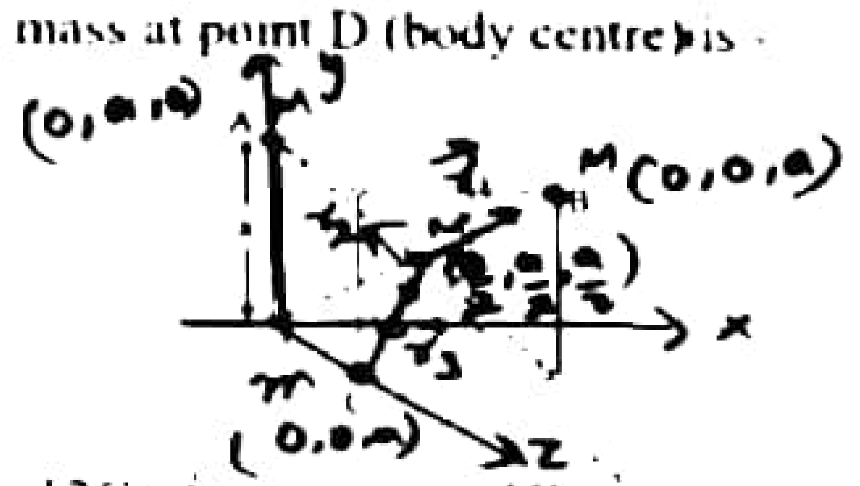
1.

Four identical masses m each are kept at points A, B, C & D shown in figure. Gravitational force on mass at point D (body centre) is -



(A) $\frac{3Gm^2}{a^2}$ (B) $\frac{12Gm^2}{a^2}$ (C) $\frac{4Gm^2}{a^2}$ (D*) $\frac{4Gm^2}{3a^2}$

Sol $\vec{F} = \frac{GM_1M_2}{r^3}\vec{r}$

$\vec{r}_1 = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$|\vec{r}| = \frac{a}{2}\sqrt{3}$

$\vec{r}_2 = -\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} - \frac{a}{2}\hat{k}$

$\vec{r}_3 = -\frac{a}{2}\hat{i} - \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}$

$\vec{F}_N = \frac{GM^2}{r^3}\left[\vec{r}_1 + \vec{r}_2 + \vec{r}_3\right]$

$= \frac{GM^2}{r^3}\left[-\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} + \frac{a}{2}\hat{k}\right]$

$|\vec{F}_N| = \frac{GM^2}{r^3} \times \left(\frac{a}{2}\sqrt{3}\right)$

$= \frac{GM^2}{r^3} \cdot r$

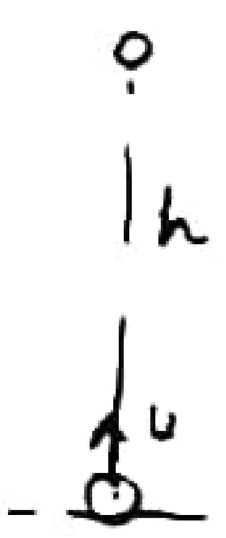
$= \frac{GM^2}{r^2}$

$F_N = \frac{4GM^2}{3a^2}$

2.

A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90m, then he will be able to throw the stone on that planet to a height-

(A) 90m (B) 40 m (C*) 100 m (D) 45 m



$u = \sqrt{2gh} = \text{const}$

$gh = \text{const}$

$g_e h_e = g_p h_p$

$\frac{M_e}{R_e^2} \cdot h_e = \frac{M_p}{R_p^2} \cdot h_p$

$1 \times 90\text{m} = \frac{1 \times 9}{10} \times h_p$

$h_p = 100\text{m}$

3.

The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is -

(A) 2R (B) $\frac{R}{\sqrt{2}}$ (C) R / 2 (D) $\sqrt{2}$R

Sol

$$g' = \frac{gR_e^2}{(R_e+h)^2}$$

$$\frac{1}{9} = \frac{R_e^2}{(R_e+h)^2}$$

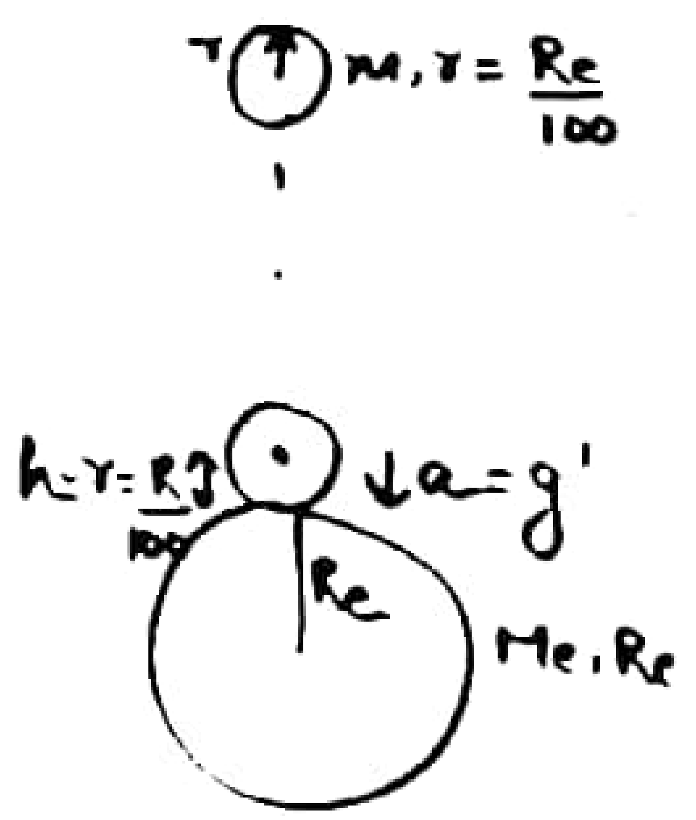
$$R_e + h = 3R_e$$

$$\boxed{h = 2R_e}$$

4.

A body of mass m and radius r falls on earth from a great height. If M is mass and R is the radius of earth while $r = \frac{R}{100}$ then the acceleration of the body when it hits the earth is: (acceleration due to gravity at earth surface is g)

(A) g (B) 0.98 g (C) $\frac{g}{0.98}$ (D) 9.8 g



$$g' = g\left(1 - \frac{2h}{R_e}\right)$$

$$g' = g\left(1 - \frac{2\times 1}{100}\right)$$

$$\boxed{g' = 0.98g}$$

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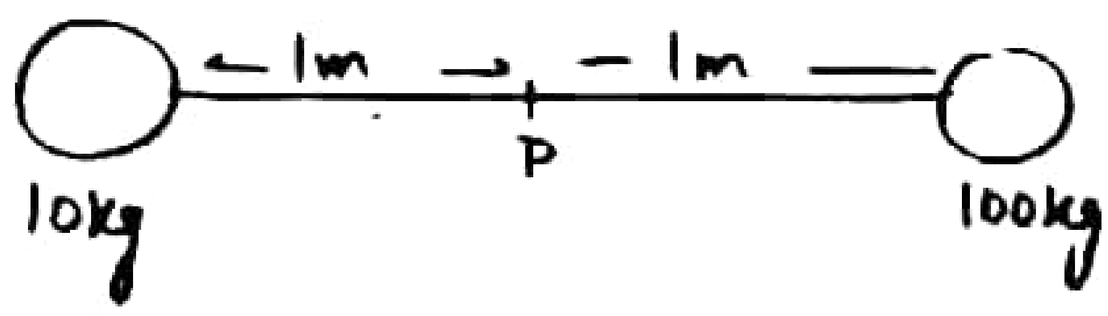
5.

Two bodies of masses 10 kg and 100 kg are separated by a distance of 2m. The gravitational potential at the mid-point of the line joining the two bodies is:

(A) -7.3×10^{-7} J/kg (B) -7.3×10^{-8} J/kg

(C) -7.3×10^{-9} J/kg (D) -7.3×10^{-6} J/kg

Sol



$$V_P = -\frac{G \times 10}{1} - \frac{G \times 100}{1}$$

$$= -110G$$

$$= -110 \times 6.67 \times 10^{-11}$$

$$= -7.3 \times 10^{-9} \text{ J/kg}$$

6.

Suppose the gravitational force varies inversely as the n^{th} power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to -

(A) $R^{\left(\frac{n+1}{2}\right)}$ (B) $R^{\left(\frac{n-1}{2}\right)}$ (C) R^n (D) $R^{\left(\frac{n-2}{2}\right)}$

Sol

$$F \propto \frac{1}{r^n}$$

$$F = \frac{k}{r^n} = mr\omega^2$$

$$\frac{k}{m \cdot r^{n+1}} = \omega^2$$

$$\omega \propto \frac{1}{r^{\frac{n+1}{2}}}$$

$$\boxed{T \propto r^{\left(\frac{n+1}{2}\right)}}$$

7.

The distance of Neptune and Saturn from the Sun are nearly 10^{13} m and 10^{12} m respectively. Their periodic times will be in the ratio -

(A) 10 (B) 100 (C*) $10\sqrt{10}$ (D) 1000

Sol $T^2 \propto r^3$

$T \propto r^{3/2}$

$$\frac{T_N}{T_s} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2}$$

$= 10^{3/2}$

$= \underline{10\sqrt{10}}$

8.

What should be the angular speed of earth, so that bodies lying on the equator may appear weightless? (R = 6400 km, g = 10 m/s²)-

(A*) 1.25×10^{-3} rad/s (B) 2.5×10^{-3} rad/s

(C) 2.0×10^{-3} rad/s (D) 3.0×10^{-3} rad/s

$\omega = 1.25 \times 10^{-3}$ r/s

Sol $g' = g - R_e\omega^2\cos^2\theta$

$g' = g - R_e\omega^2$

at equator body in weightless $g' = 0$

$g - R_e\omega^2 = 0 \Rightarrow \omega = \sqrt{\frac{g}{R_e}}$

$\omega = \sqrt{\frac{10}{64 \times 10^5}}$

$\omega = \sqrt{\frac{1}{64 \times 10^4}}$

$\omega = \frac{1}{800}$ rad/s

$\omega = \frac{10}{8} \times 10^{-3}$ rad/s

9.

Statement – 1: When a planet approaches the point which is farthest from the Sun, its orbital speed decreases. correct

Statement – 2: Work done on the planet by the gravitational force exerted by the Sun is negative correct

(1✓) Both Statement – 1 and Statement – 2 are true

(2) Statement – 1 is true and Statement – 2 is false

(3) Statement – 1 is False and Statement – 2 is true

(4) Both Statement – 1 and Statement – 2 are False

$W_C = -\Delta U$

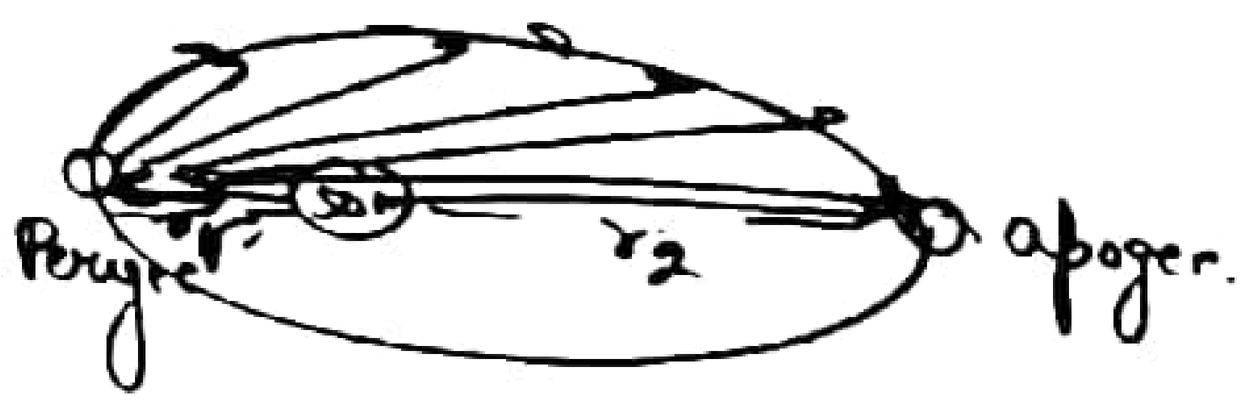
$W_C = -[U_f - U_i]$

$= U_i - U_f$

$= -\frac{GM_1M_2}{r_1} + \frac{GM_1M_2}{r_2}$

$W_C = GM_1M_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$r_2 > r_1$, $\frac{1}{r_2} < \frac{1}{r_1}$ $\Rightarrow W_C = \underline{-ve}$



10.

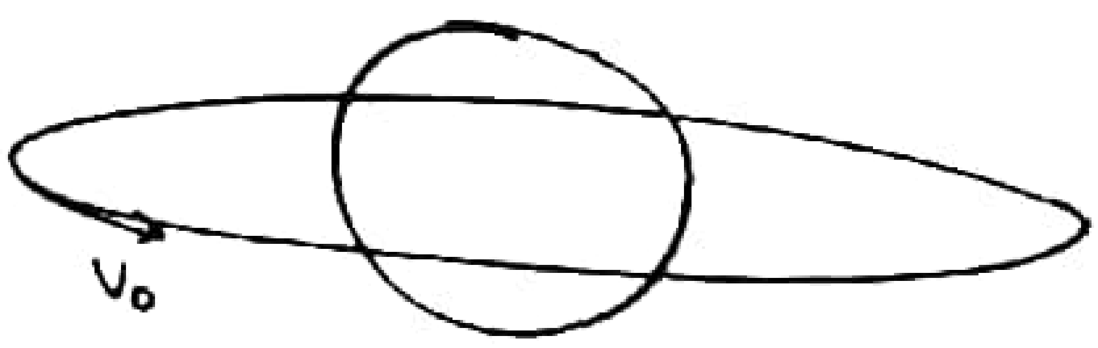
A Geostationary satellite is revolving around the earth. To make it escape from gravitational field of earth, its velocity must be increased -

(A) 100 % (B✓) 41.4% (C) 50% (D) 59.6%

% change in vel $= \left(\frac{V_f - V_i}{V_i}\right) \times 100$

$= \left(\frac{\sqrt{2} - 1}{1}\right) \times 100$

$= \underline{41.4\%}$



$V_e = \sqrt{2}\, V_0$

11.

A comet travels around the sun in elliptical orbit. Its mass is 10^8 kg when 2.5×10^{11} m farthest away its speed is 2×10^4 ms^{-1}. Find the change in KE when it has reached 5×10^{10} m closest from the sun -

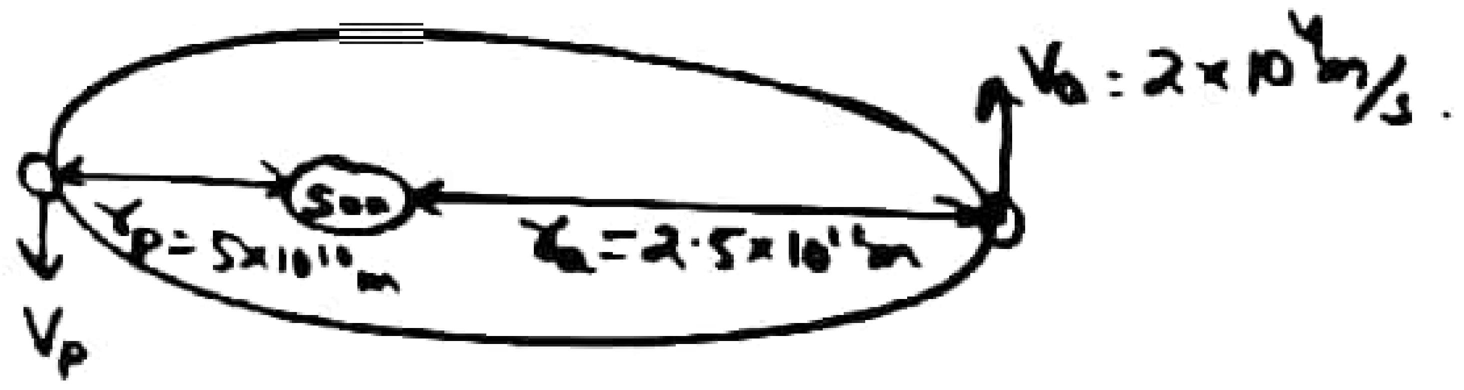
(A) 38×10^8 J (B) 48×10^{16} J (C) 58×10^8 J (D) 56×10^8 J

$$\Delta K = \frac{1}{2} m \left[v_p^2 - v_a^2 \right]$$

$$= \frac{1}{2} \times 10^8 \left[10^8\right] \left[10^2 - 2^2 \right]$$

$$= 10^{16} \times \frac{96}{2}$$

$$\Delta k = 48 \times 10^{16} \text{ J}$$



$$V_p r_p = V_a r_a$$

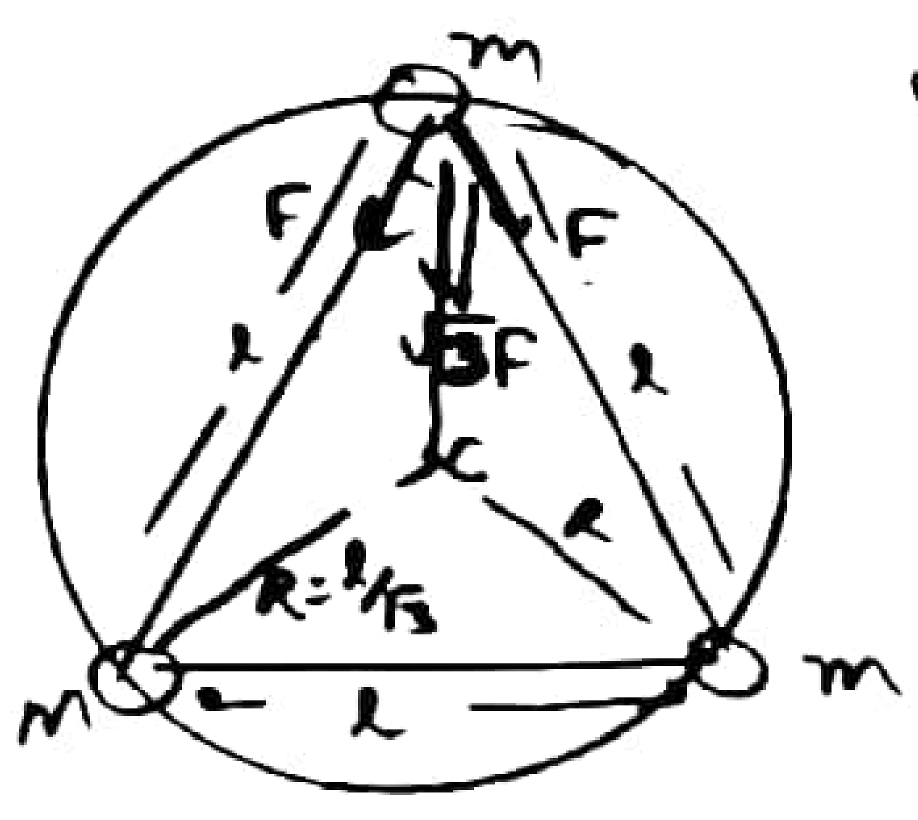
$$V_p = 2 \times 10^4 \times \frac{\cancel{25} \times 10^{10}}{5 \times 10^{10}}$$

$$V_p = 10^5 \text{ m/s}$$

12.

Three particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is -

(A) $V = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (B) $V = \sqrt{\frac{Gm}{2R}}$ (C) $V = \sqrt{\frac{Gm}{\sqrt{3}R}}$ (D) $V = \sqrt{\frac{4Gm}{R}}$



$$\sqrt{3}F = \frac{mv^2}{R}$$

$$\sqrt{3}\frac{Gm^2}{l^2} = \frac{mv^2}{R}$$

$$\frac{\sqrt{3}Gm}{3R^2} = \frac{v^2}{R}$$

$$v = \sqrt{\frac{Gm}{\sqrt{3}R}}$$

13.

The radii of two planets are respectively R_1 & R_2 and their densities are respectively ρ_1 & ρ_2. The ratio of the acceleration due to gravity at their surface is -

(A) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (B) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$

(C) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$ (D*) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

Sol

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\rho \cdot \frac{4}{3}\pi R^3$$

$$g = G\rho \cdot \frac{4}{3}\pi R$$

$$g \propto \rho R$$

$$\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2}\frac{R_1}{R_2}.$$

14.

The moon's radius is 1/4 th that of the earth & its mass is $\frac{1}{80}$ times that of earth. if g represents the acceleration due to gravity on the surface of earth, that on the surface of moon is -

(A) $\frac{g}{4}$ (B*) $\frac{g}{5}$ (C) $\frac{g}{6}$ (D) $\frac{g}{8}$

Sol

$$\frac{g_M}{g} = \frac{M_M}{M_e}\frac{R_e^2}{R_M^2}.$$

$$\frac{g_e}{g} = \frac{1}{80} \times 16.$$

$$g_e = g/5$$

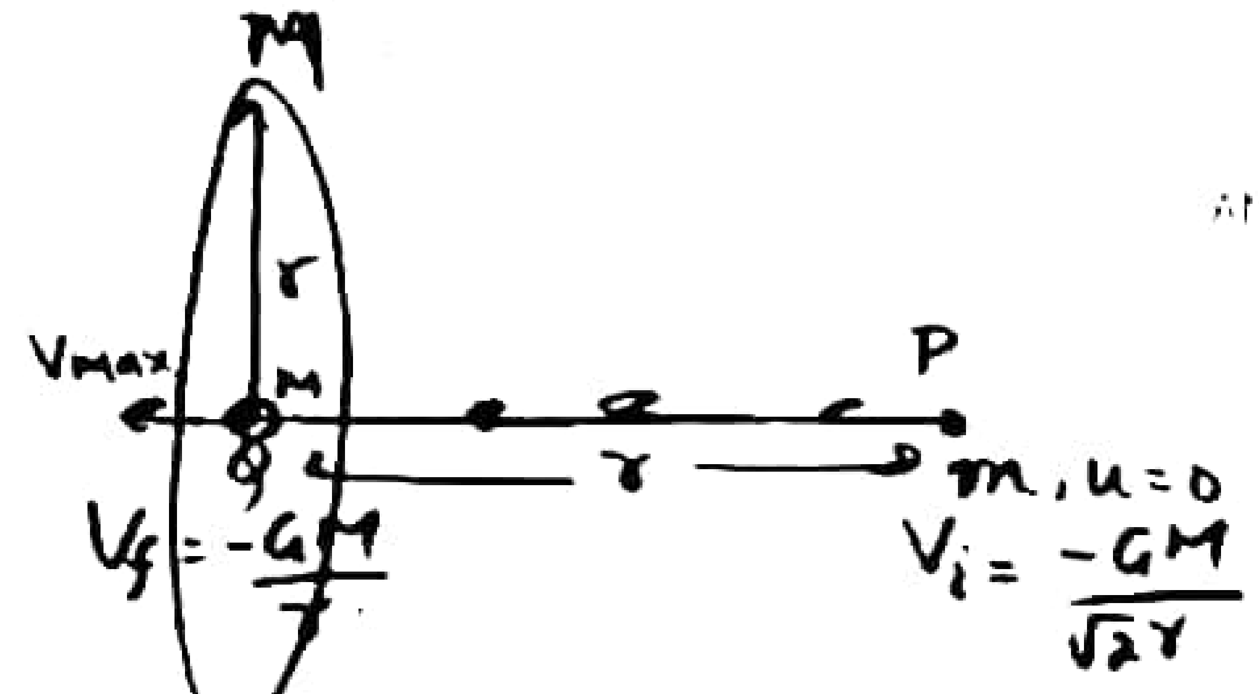
15.

Consider a particle of mass m on the axis of a ring of mass M & radius r, at a distance r from the centre of ring, this particle moving under the gravitational attraction of the ring, reaches the centre of the ring. The velocity of the particle at the centre of the ring will be -

(A) $\sqrt{\frac{GM}{r}}$ (B) $\sqrt{\frac{2GM}{r}}$

(C) $\sqrt{\frac{2GM}{r(\sqrt{2}-1)}}$ (D) ✓ $\sqrt{\frac{2GM}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$

Sol



$K_i + U_i = K_f + U_f$

$K_i + mV_i = K_f + mV_f$

$0 + m[V_i - V_f] = K_f$

$\cancel{m}\left[\frac{-GM}{\sqrt{2}r} + \frac{GM}{r}\right] = \frac{1}{2}\cancel{m}v^2$

$\sqrt{\frac{2GM}{r}\left[1-\frac{1}{\sqrt{2}}\right]} = v$

16.

A satellite is in a circular equatorial orbit of radius 7000 km around the Earth. If it is transferred to a circular orbit of double the radius

	Column I		Column II
(A)	Angular momentum → p	(p)	Increases
(B)	Area of Earth covered by satellite signal → p	(q)	Decreases
(C)	Potential energy → p, s	(r)	Becomes double
(D)	Kinetic energy → q, s	(s)	Becomes half

(1✓) a → p; b → p; c → p, s; d → q, s
(2) a → q : b → r : c → r, s ; d → p, s
(3) a → s ; b → p ; c → q, s ; d → p, q
(4) a → r ; b → r ; c → p, r ; d → p, q

$K = \frac{1}{2}mv^2$

$K_i = \frac{GMm}{2R}$

$K_f = \frac{GMm}{4R}$

$L = mvR = m\sqrt{\frac{GM}{R}}\,R$

$L = m\sqrt{GMR}$

$U_i = -\frac{GM_1M_2}{R}$ $U_f > U_i$

$U_f = -\frac{GM_1M_2}{2R}$

17.

A mass m is raised from a distance 2R from surface of earth to 3R. Work done to do so against gravity will -

(A) $\frac{mgR}{10}$ (B) $\frac{mgR}{11}$ (C) $\frac{mgR}{12}$ (D) $\frac{mgR}{14}$

Sol $W_g = -\Delta U$

$W_{against} = +\Delta U$

$= GM_1M_2\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= GM_em\left[\frac{1}{3R} - \frac{1}{4R}\right]$

$= \frac{GM_em}{12R}$

$W_{against} = \frac{gR_e^2 \cdot m}{12R_e}$

$= \frac{mgR}{12}$

18.

A satellite of the sun is in circular orbit around the sun, midway between the sun and the earth. Then:

(A) The period of the satellite is nearly 229 days

(B) The period of the satellite is nearly 129 days

(C) The speed of the satellite equal the escape velocity of the earth

(D) The acceleration of the satellite is four times the acceleration of the earth.

$T_E^2 \propto R^3$

$T_s^2 \propto \left(\frac{R}{2}\right)^3$

$\frac{365}{T_s} = (2)^{3/2}$

$= 2\sqrt{2}$

$T_s = \frac{365}{2\sqrt{2}} \approx 129 \text{ days}$

19.

The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth

(a) Is the same
(b) Is smaller
(c) Is greater
(d) Varies with its phase

20.

A weight is suspended from the ceiling of a lift by a spring balance. When the lift is stationary the spring balance reads W. If the lift suddenly falls freely under gravity, the reading on the spring balance will be

(a) W (b) $2W$ (c) $W/2$ (d) 0

21.

A pendulum clock is set to give correct time at the sea level. This clock is moved to hill station at an altitude of 2500 m above the sea level. In order to keep correct time of the hill station, the length of the pendulum

(a) Has to be reduced
(b) Has to be increased
(c) Needs no adjustment
(d) Needs no adjustment but its mass has to be increased

$$T = 2\pi\sqrt{\frac{l(\downarrow)}{g(\downarrow)}}$$

22.

Statement – 1: The acceleration of a particle near the Earth surface differs slightly from the gravitational acceleration.

Statement – 2: The Earth is not a uniform sphere and because the Earth rotates.

(1) Both Statement – 1 and Statement – 2 are true
(2) Statement – 1 is true and Statement – 2 is false
(3) Statement – 1 is False and Statement – 2 is true
(4) Both Statement – 1 and Statement – 2 are False

23.

The ratio of the radius of the earth to that of the moon is 10. The ratio of acceleration due to gravity on the earth and on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is

(a) 10 (b) 6 (c*) Nearly 8 (d) 1.66

Sol $\frac{R_E}{R_M} = 10$ $\frac{g_e}{g_m} = 6$

$$\frac{(v_e)_{Earth}}{(v_e)_{Moon}} = \sqrt{\frac{g_e}{g_m} \times \frac{R_e}{R_m}}$$

$$= \sqrt{6 \times 10} = \sqrt{60} \simeq 8$$

24.

Two identical satellites A and B are circulating round the earth at the height of R and $2R$ respectively. (where R is radius of the earth). The ratio of kinetic energy of A to that of B is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d*) $\frac{3}{2}$

$$K = \frac{GM_1M_2}{2R}.$$

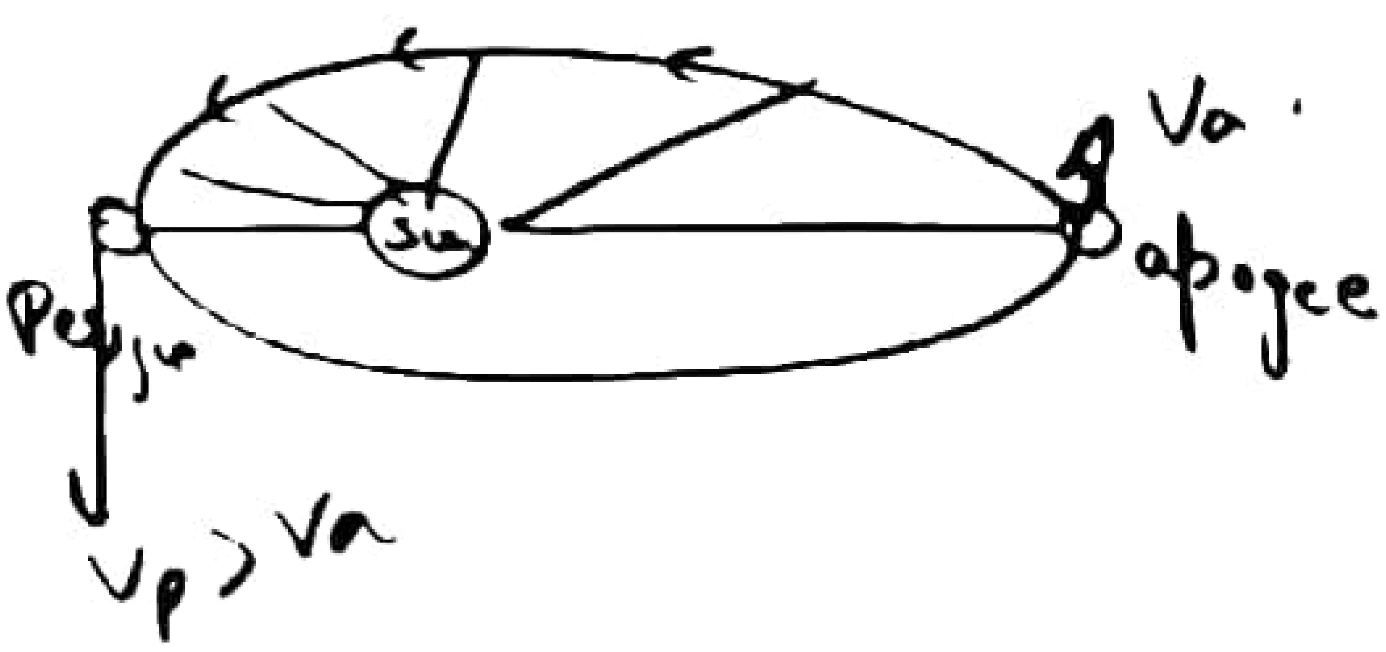
$$\frac{K_1}{K_2} = \frac{R_2}{R_1} = \frac{3R}{2R} = \frac{3}{2}.$$

25.

[illegible]

	Column-I		Column-II
(A)	Speed of planet $\rightarrow r$	(p)	Remains same
(B)	Distance of planet from centre of Sun $\rightarrow q$	(q)	Decreases
(C)	Potential energy $\rightarrow q$	(r)	Increases
(D)	Angular momentum about centre of Sun $\rightarrow p$	(s)	Cannot say

✓(1) A→r, B→q, C→q, D→p (2) A→q, B→r, C→s, D→p
(3) A→p, B→s, C→q, D→r (4) A→s, B→p, C→q, D→r



26.

A geostationary satellite is orbiting the earth at a height of 5R above that surface of the earth, R being the radius of the earth. The time period of another satellite in hours at a height of 2R from the surface of the earth is:

(1) 5 (2) 10 ✓(3*) $6\sqrt{2}$ (4) $\frac{6}{\sqrt{2}}$

$$T \propto R^{3/2}$$

$$\frac{24}{T} = \left(\frac{6R}{3R}\right)^{3/2}$$

$$\frac{24}{T} = 2\sqrt{2}$$

$$T = \frac{12}{\sqrt{2}} \text{ hr}$$

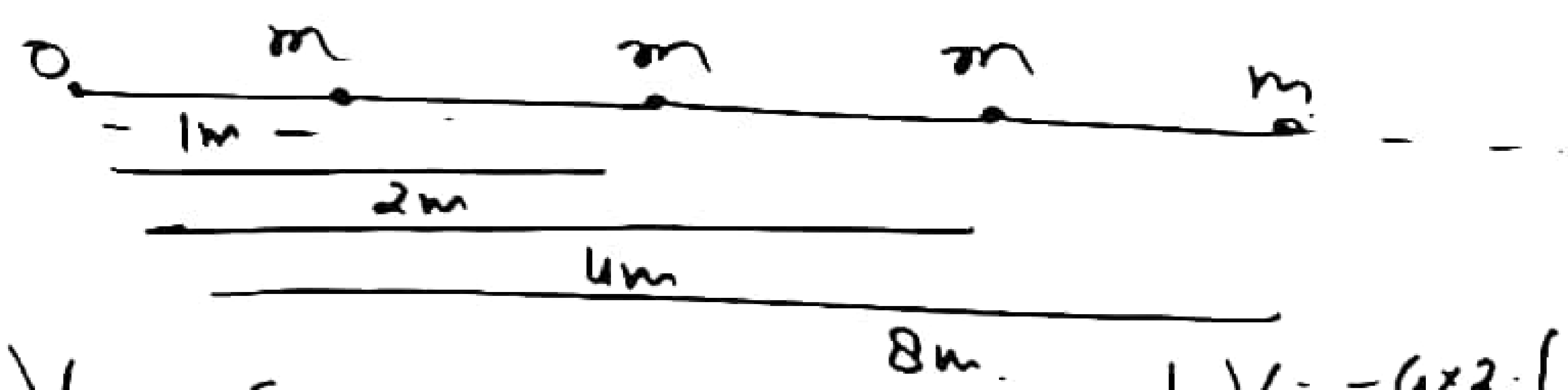
$$= 6\sqrt{2} \text{ hr}.$$

27.

Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1m, 2m, 4m, 8m ... Respectively, from the origin. The resulting gravitational potential due to this system at the origin will be:

(1) $-\frac{8}{3}G$ (2) $-\frac{4}{3}G$ (3) $-4G$ (4) $-G$

Sol



$$V_0 = -\frac{GM}{1} - \frac{GM}{2} - \frac{GM}{4} - - -$$

$$= -GM\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+ - -\right)$$

$$V = -G\times 2\left[\frac{1}{1-\frac{1}{2}}\right]$$

$$= \underline{-4G}$$

28.

A block hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

(1) 10^{-9} m (2) 10^{-6} m (3) 10^{-2} m (4) 100 m

Sol

$$V_e = c$$

$$\sqrt{\frac{2GM}{R}} = c$$

$$\frac{2GM}{R} = c^2$$

$$R = \frac{2GM}{c^2}$$

$$R = \frac{2\times\frac{20}{3}\times10^{-11}\times\cancel{6}^{\,2}\times10^{24}}{3\,\cancel{9}\times10^{16}}$$

$$R = \frac{\cancel{80}^{\,9}}{\cancel{9}}\times10^{-3}\ m$$

$$R = 9\times10^{-3}\ m$$

$$R \approx \underline{10^{-2}\ m}$$

29.

The gravitational field in a region is 10 N/kg $(\hat{i} - \hat{j})$. Then the work done by gravitational force to shift slowly a particle of mass 1kg from point (1m, 1m) to a point (2m, −2m) is-

(A) 10 J (B) −10J (C) −40J (D*) + 40 J

Sol $\vec{E} = 10(\hat{i} - \hat{j})$

$m = 1\,kg$.

$\vec{\Delta r} = 1\hat{i} - 3\hat{j}$

$W = \vec{F} \cdot \vec{\Delta r}$

$= m(\vec{E} \cdot \vec{\Delta r})$

$= 1 \times (10)(1+3)$

$W = 40\,J$

30.

A body is projected from the surface of earth with a velocity $2v_e$ where v_e is the escape velocity. The velocity of the body when it escapes the gravitational field of the earth is:

(A) $\sqrt{2}v_e$ (B*) $\sqrt{3}v_e$ (C) $\sqrt{7}v_e$ (D) $\sqrt{11}v_e$

v_{∞}

∞

$v_i = 2v_e$

$n = 2$

$v_{\infty} = \left[\sqrt{(n^2 - 1)}\right] v_e$

$v_{\infty} = \sqrt{3}v_e$

31.

Statement – 1: Two satellites A & B are in the same orbit around the Earth. B being behind A. B cannot overtake A by increasing its speed. Correct
Statement – 2: It will then go into a different orbit. Correct

(1) Both Statement – 1 and Statement – 2 are true

(2) Statement – 1 is true and Statement – 2 is false

(3) Statement – 1 is False and Statement – 2 is true

(4) Both Statement – 1 and Statement – 2 are False

32.

Acceleration due to gravity at earth's surface is $10\ ms^{-2}$. The value of acceleration due to gravity at the surface of a planet of mass $\frac{1}{5}$th and radius $\frac{1}{2}$ of the earth is -

(A) $4\ ms^{-2}$ (B) $6\ ms^{-2}$ (C) $8\ ms^{-2}$ (D) $12\ ms^{-2}$

$$\frac{g_p}{g} = \frac{M_p}{M_e} \times \left(\frac{R_e}{R_p}\right)^2$$

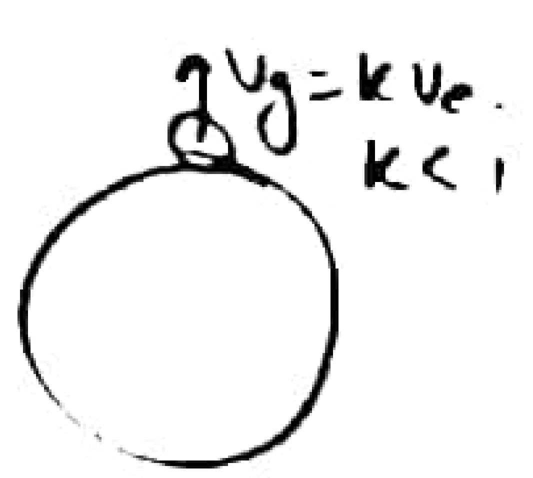
$$= \frac{1}{5} \times 4$$

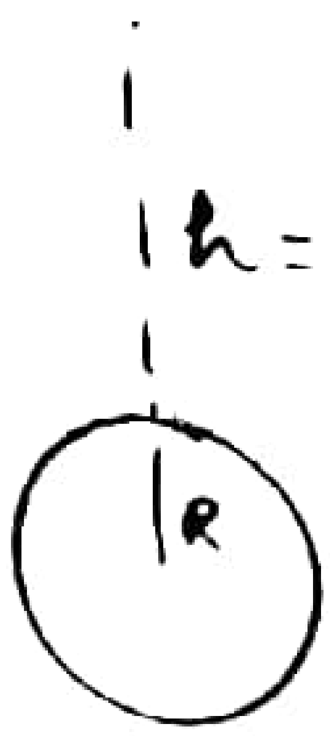
$$g_p = \frac{4}{5} \times 10 = 8\ m/s^2$$

33.

A projectile is fired vertically upwards from the surface of the earth with a velocity kv_e, where v_e is the escape velocity and $k < 1$. If R is the radius of the earth, the maximum height to which it will rise measured from the centre of earth will be - (Neglect air resistance)

(A) $\frac{1-k^2}{R}$ (B) $\frac{R}{1-k^2}$ (C) $R(1-k^2)$ (D) $\frac{R}{1+k^2}$





$h = \frac{n^2 R_e}{1-n^2} = \frac{k^2 R_e}{1-k^2}$

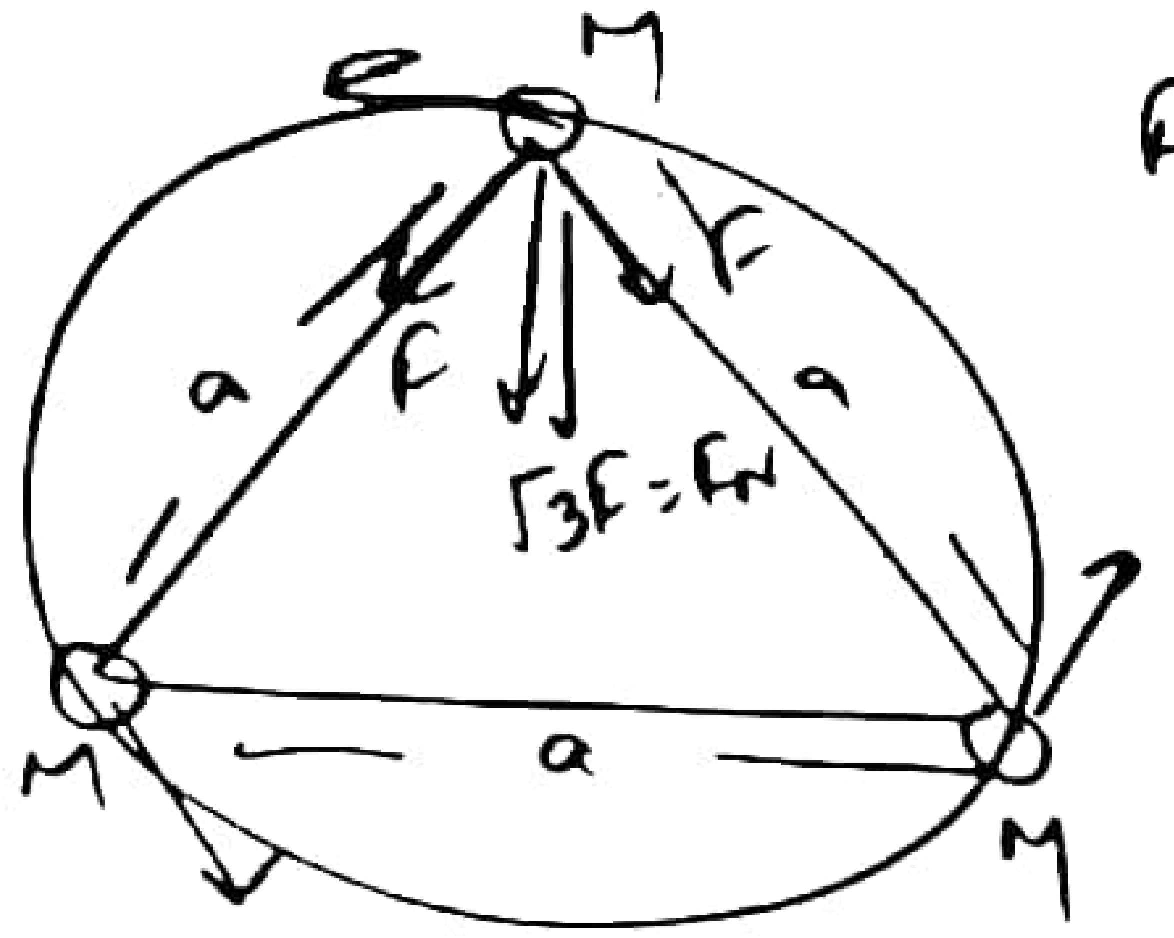
From Centre $= h + R_e$

$= \frac{R_e}{1-k^2}$

34.

Three particles ~~each of~~ mass M are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation a. What is the net gravitational force on one of the particles?

(A) $\frac{GM^2}{a^2}$ (B) $\sqrt{2}\frac{GM^2}{a^2}$ (C) $\sqrt{3}\frac{GM^2}{a^2}$ (D) $\frac{2GM^2}{a^2}$



$F_N = \sqrt{3} F$

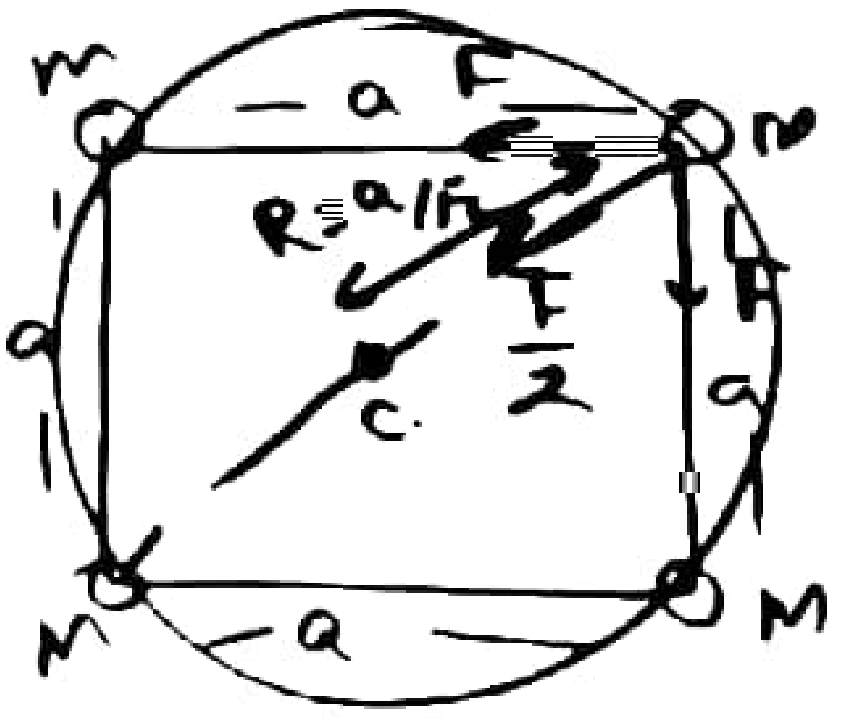
$= \sqrt{3}\frac{GM^2}{a^2}$

35.

A, B, C and D are four masses each of mass M lying on the vertices of a square of side 'a'. They always move along a common circle with velocity v. Find v so that they always remain on the vertices of the square -

(A) $\sqrt{\frac{GM(2\sqrt{2}+1)}{2\sqrt{2}a}}$ (B) $\sqrt{\frac{GM(\sqrt{2}+1)}{2a}}$

(C) $\sqrt{\frac{GM\sqrt{2}(2+1)}{a}}$ (D) none



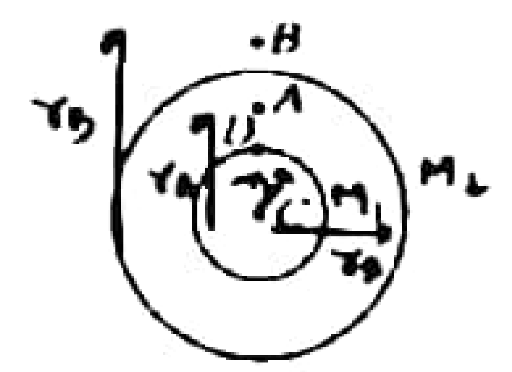
$F_N = \sqrt{2}F + \frac{F}{2}$

$F_N = \frac{(2\sqrt{2}+1)}{2}\frac{GM^2}{a^2} = \frac{Mv^2}{a/\sqrt{2}}$.

$v = \sqrt{\frac{(2\sqrt{2}+1)}{2\sqrt{2}}\frac{GM}{a}}$.

36.

Two concentric spherical shells are as shown in figure.



	Column–I		Column–II
(A)	Potential at A → q	(p)	greater than B
(B)	Gravitational field at A → t	(q)	less than B
(C)	As one moves from C to D → r	(r)	potential remains constant
(D)	As one moves from D to A → s	(s)	gravitational field decreases
		(t)	None

(1) A→q, B→t, C→r, D→s (2) A→r, B→q, C→p, D→t
(3) A→s, B→p, C→q, D→r (4) A→t, B→r, C→p, D→s

p) $V_A = -\frac{GM_1}{r_A} - \frac{GM_2}{r_2}$

$V_B = -\frac{GM_1}{r_B} - \frac{GM_2}{r_B}$

$r_B > r_A$

$r_B > r_2$

$V_B > V_A$

B) $E_A = \frac{GM_1}{r_A^2} + 0$

$E_B = \frac{GM_1}{r_B^2} + \frac{GM_2}{r_B^2}$

$V_D = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2}$

$V_A = -\frac{GM_1}{r_A} - \frac{GM_2}{r_2}$

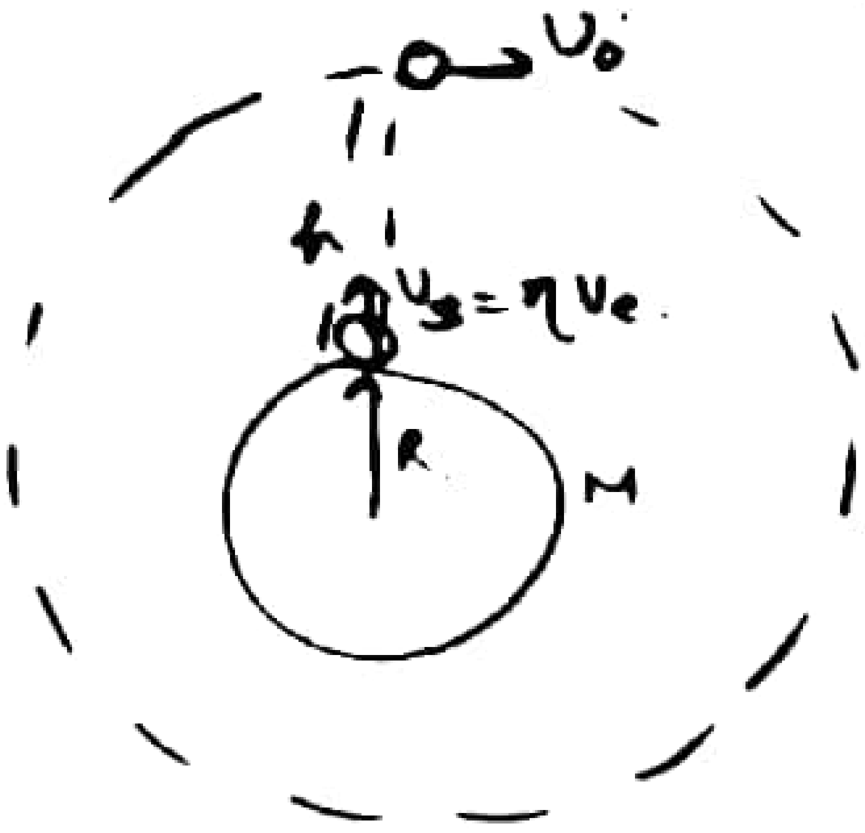
$E_D = \frac{GM_1}{r_1^2} + 0$

$E_A = \frac{GM_1}{r_A^2} + 0$

37.

A projectile is fired from the surface of earth of radius R with a velocity $\eta\, v_e$ where v_e is the escape velocity and $\eta < 1$. Neglecting air resistance, the orbital velocity of projectile is -

(A) $v_e\sqrt{1-\eta^2}$ (B) $v_e\sqrt{\frac{\eta^2}{5}}$ (C) $\frac{2}{5}v_e\sqrt{\eta}$ (D) $\frac{2\eta}{5}v_e$



$v_0 = \sqrt{\frac{GM_e}{(R+h)}}$

$K_i + U_i = K_f + U_f$

$\frac{1}{2} m\eta^2 v_e^2 - \frac{GM_eM}{R} = \frac{1}{2} mv_0^2 - \frac{GM_eM}{(R+h)}$

$m\eta^2 v_e^2 - \frac{2GM_e}{R_e} m = mv_0^2 - \frac{2GM_eM}{(R_e+h)}$

$m\eta^2 v_e^2 - mv_e^2 = mv_0^2 - 2mv_0^2$

$(\eta^2-1)v_e^2 = -v_0^2$

$\boxed{v_0 = \sqrt{(1-\eta^2)}\; v_e}$

38.

The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio 3: 4. If the two shells coalesce into single one such that surface charge density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to -

(A) 3: 2 (B) 4: 3 (C*) 5: 3 (D) 5: 6

$V = -\frac{GM}{r} = -\frac{G\cdot\sigma\cdot 4\pi R^2}{R}$

$V = -G\sigma 4\pi R$

$\frac{V_1}{V_2} = \frac{3}{4}$

$\Rightarrow \frac{R_1}{R_2} = \frac{3}{4}$ $\left.\begin{array}{l} R_1 = 3R \\ R_2 = 4R \end{array}\right\}$

$m_1 + m_2 = m$

$\sigma 4\pi r_1^2 + \sigma 4\pi r_2^2 = \sigma 4\pi r'^2$

$r'^2 = r_1^2 + r_2^2$

$r' = 5R$

$\frac{V'}{V_1} = \frac{r'}{r_1} = \frac{5}{3}$

39.

Statement – 1: A person feels weightlessness in an artificial satellite of the Earth. However a person on the Moon (natural satellite) feels his weight. Correct

Statement – 2: Artificial satellite is a freely falling body and on the Moon surface, the weight is mainly due to Moon's gravitational attraction. Correct

(1*) Both Statement – 1 and Statement – 2 are true

(2) Statement – 1 is true and Statement – 2 is false

(3) Statement – 1 is False and Statement – 2 is true

(4) Both Statement – 1 and Statement – 2 are False

40.

What should be the angular velocity of earth about its own axis so that the weight of the body at equator would become ¾ th of its present value-

(A) $\frac{1}{400}$ rad/s (B) $\frac{1}{800}$ rad/s (C*) $\frac{1}{1600}$ rad/s (D) $\frac{1}{3200}$ rad/s

Sol $g' = g - R_e\omega^2\cos^2 0$

$\frac{3}{4}g = g - R_e\omega^2$

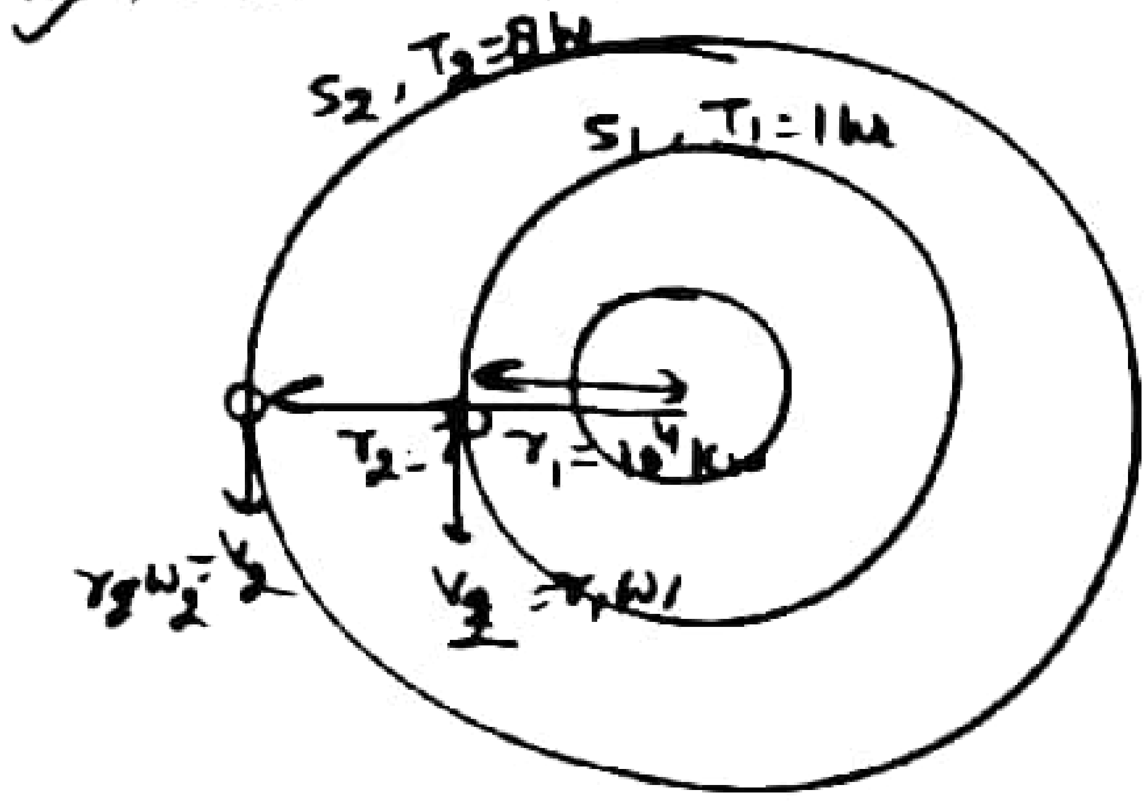
$\omega = \sqrt{\frac{g}{4R_e}} = \sqrt{\frac{10}{4\times64\times10^5}}$

$= \frac{1}{1600}$ rad/s.

41.

Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hour respectively. The radius of the orbit of S_1 is 10^4 km. The speed of S_2 relative to S_1 when they are closet (in kmh^{-1}) is

(A) $-10^4\pi$ (B) $2 \times 10^4\pi$ (C) $\frac{1}{2} \times 10^4\pi$ (D) $4 \times 10^4\pi$



$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$R_2 = \left(\frac{T_2}{T_1}\right)^{2/3} R_1$$

$$R_2 = \left[\frac{8}{1}\right]^{2/3} \times 10^4$$

$$R_2 = 4 \times 10^4 \text{ km}$$

$$v_{2/1} = v_2 - v_1 = R_2\omega_2 - R_1\omega_1 = 4\times10^4 \times \frac{2\pi}{8} - 10^4 \times \frac{2\pi}{1}$$

$$v_{2/1} = 2\pi\times10^4\left[\frac{1}{2} - 1\right]$$

$$v_{2/1} = -\pi\times10^4 \text{ km/h}$$

42.

At a given place where acceleration due to gravity is 'g' m sec^{-2}, a sphere of lead of density 'd' kg m^{-3} is gently released in a column of liquid of density 'ρ' kg m^{-3}. If $d > \rho$, the sphere will

(a) Fall vertically with an acceleration 'g' m sec^{-2}

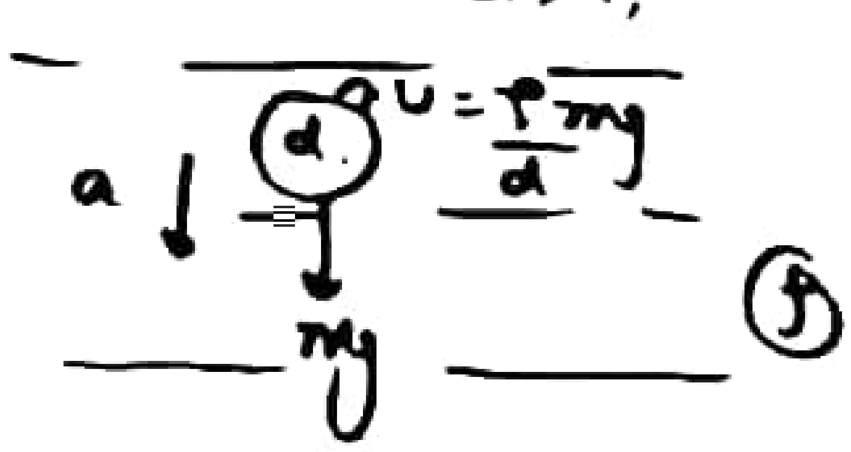
(b) Fall vertically with no acceleration

(c) Fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$

(d) Fall vertically with an acceleration $g\left(\frac{\rho}{d}\right)$

Sol

$d > \rho$,



$$a = g\left(1 - \frac{\rho}{d}\right)$$

43.

The mean radius of the earth's orbit around the sun is 1.5×10^{11}. The mean radius of the orbit of mercury round the sun is 6×10^{10} m. The mercury will rotate around the sun in

(a) A year (b) Nearly 4 years

✓(c) Nearly $\frac{1}{4}$ year (d) 2.5 years

$$\frac{T_M}{T_E} = \left(\frac{R_M}{R_E}\right)^{3/2}$$

$$T_M = 1\,yr\left(\frac{6\times10^{10}}{1.5\times10^{10}}\right)^{3/2}$$

$$= 1\,yr\left(\frac{2}{5}\right)^{3/2}$$

$$= \frac{2}{5}\sqrt{\frac{2}{5}} = \frac{2}{5}\times\frac{1}{1.4\,0.7} = \frac{1}{3.5}\,yr.$$

44.

Match the following

	Column-I		Column-II
(A)	Kinetic energy of a particle in gravitational field is increasing → s	(p)	work done by gravitational force should be positive
(B)	Potential energy of a particle in gravitational field is increasing → r	(q)	work done by external force should be non-zero
(C)	Mechanical energy of a particle in gravitational field is increasing → q	(r)	work done by gravitational force should be negative
		(s)	cannot say anything

✓(1) A→s, B→r, C→q (2) A→p, B→p, C→r

(3) A→q, B→s, C→p (4) A→p, B→q, C→r

A) $W = \Delta K$

$W_g + W_c = \Delta K = +w.$

B) $W_c = -\Delta U$

$\Delta U = +w$

$W_c = -w$

$W_g = \Delta K + \Delta U$

$= \Delta E = +ve$

45.

A satellite is moving round the earth. In order to make it move to infinity, its velocity must be increased by

(1) 20% (2) it is impossible to do so

(3) 82.8% (4*) 41.4%

Sol V_0 , $\sqrt{2}V_0$

$$\% \text{ Inc} = \frac{\sqrt{2}-1}{1} \times 100$$

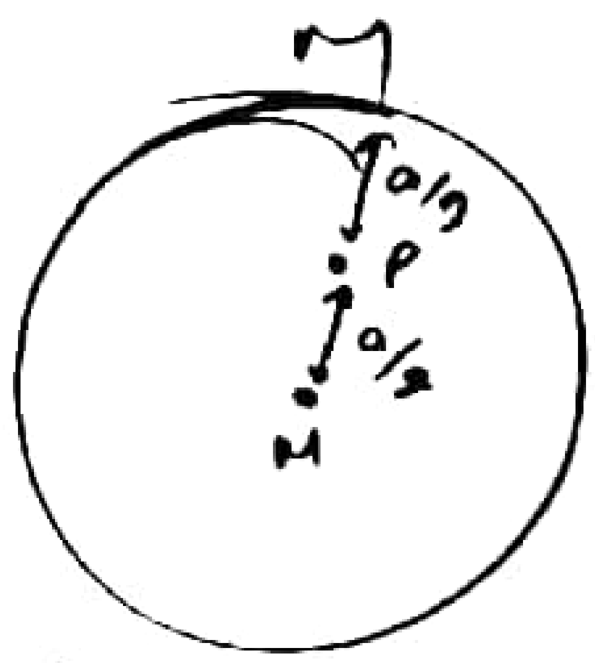
$$= \underline{41.4\,\%}$$

46.

A particle of mass M is situated at the center of a spherical shell of same mass and radius a. The gravitational potential at a point situated at distance a/2 from the centre, will be

(1*) $\frac{3GM}{a}$ (2) $-\frac{2GM}{a}$ (3) $-\frac{GM}{a}$ (4) $-\frac{4GM}{a}$

Sol



$$V_P : -\frac{2GM}{a} - \frac{GM}{a} = -\frac{3GM}{a}$$

47.

The radii of circular orbits of two satellites A and B of the earth, are 4R and R, respectively. If the speed of satellite A is 3V, then the speed of satellite B will be

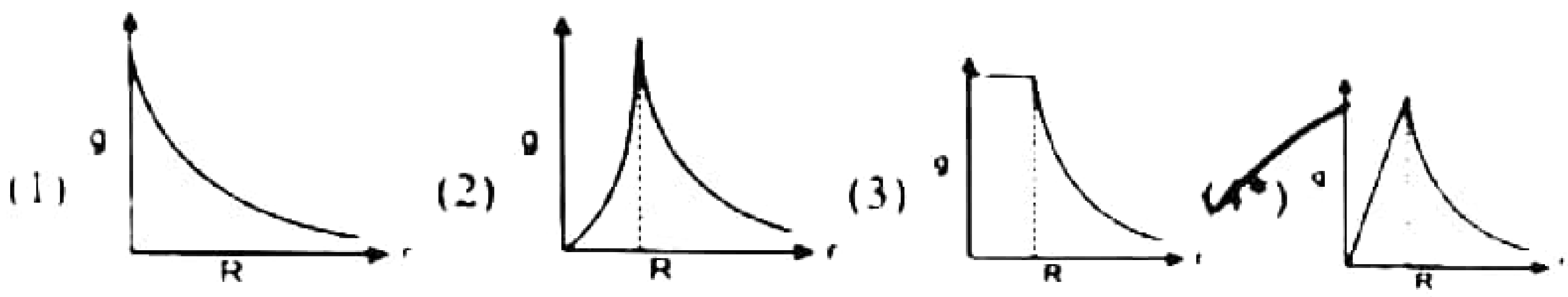
(1) $\frac{3V}{4}$ (2) 6V (3) 12 V (4) $\frac{3V}{2}$

$$\frac{3V}{V_B} = \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$V_B = 6V$$

48.

The dependence of acceleration due to gravity g on the distance r from the centre of the earth, assumed to be a sphere of radius R of uniform density is as shown in figures below. The correct figure is.



49.

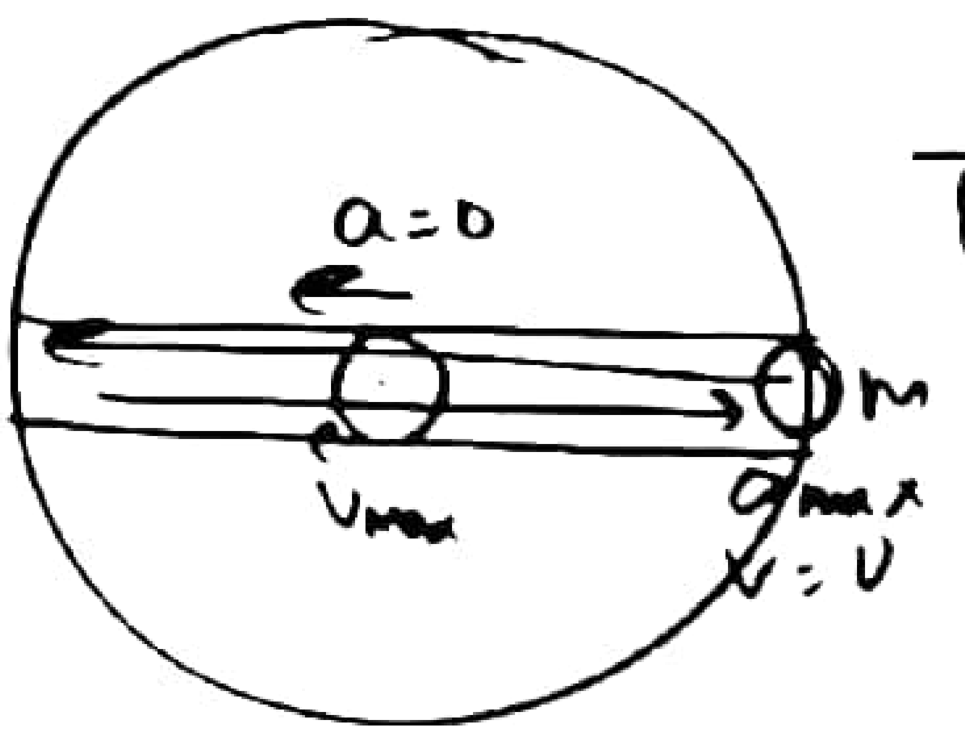
Suppose a tunnel is dug along the diameter of the earth. A particle is dropped from a point directly above the tunnel. If the earth's density is assumed to be uniform and the friction is neglected, then wrong statement is

(1) particle will have maximum speed when passing through the centre of the earth.

(2) Acceleration of particle will be the maximum just at the point of release

(3) Particle will have harmonic oscillation

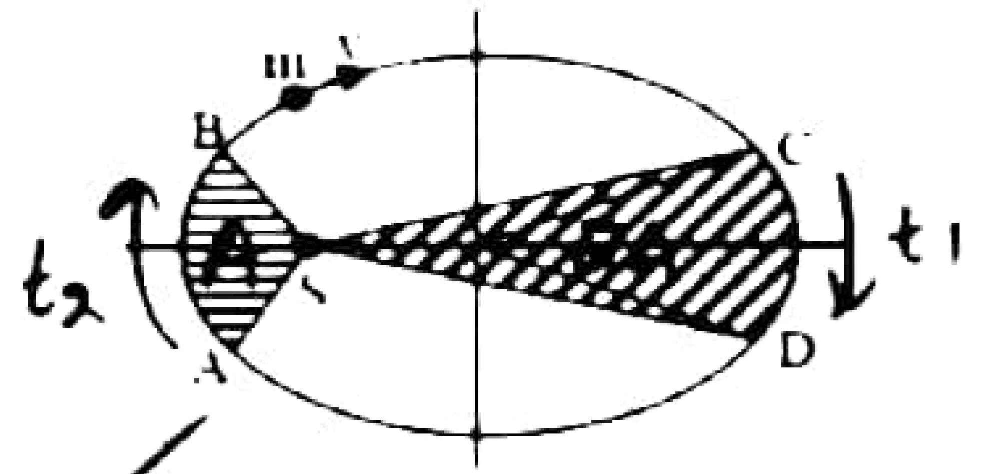
(4) Particle will drop to the centre of the earth



$$T = 2\pi\sqrt{\frac{R_e}{g}}$$

50.

The figure shows elliptical orbit of a planet m about the sun S. The shaded area SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from C to D and t_2 is the time to move from A to B then



(1) $t_1 = 4t_2$ (2) $t_1 = 2t_2$ (3) $t_1 = t_2$ (4) $t_1 > t_2$

$$\frac{\Delta A}{\Delta t} = \text{Cont}$$

$$\frac{2A}{t_1} = \frac{A}{t_2}$$

$$t_1 = 2t_2$$